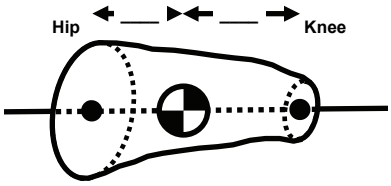
Inertial Properties

2D Example

1. Use the proportions in Table 1 (Dempster, 1955) to calculate the thigh mass for a 90.0 kg person.

   m_{thigh} = _____ × m_{total} = _____ × 90.0 = _____ kg

2. Use the proportions in Table 1 (Dempster, 1955) to calculate the thigh center of mass given the thigh length is 0.463 m.



   Distance from hip center:
   r_{thigh} = _____ × l_{thigh} = _____ × 0.463 = _____ m

3. Use the proportions in Table 1 (Dempster, 1955) to calculate the thigh moment of inertia about its center of mass for a 90.0 kg person given the thigh length is 0.463 m.

   $k_{zz}^{thigh/cm}$ = _____ × l_{thigh} = _____ × 0.463 = _____ m

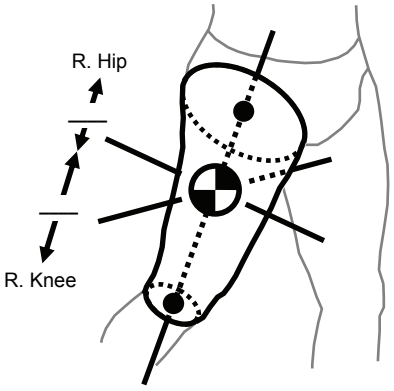
   $I_{zz}^{thigh/cm} = m_{thigh} \times (k_{zz}^{thigh/cm})^2$ = _____ × _____ = _____ kg·m^2

3D Example

1. Use the regression equations in Table 2 (Vaughan *et al*., 1992) to calculate the thigh mass for a 90.0 kg person given the thigh length is 0.463 m and mid-thigh circumference is 0.445 m.

   m_{thigh} = _____ × m_{total} + _____ × [l_{thigh} × (c_{thigh})^2] – _____
   m_{thigh} = _____ × 90.0 + _____ × [0.463 × (0.445)^2] – _____ = _____ kg

2. Use the proportions in Table 2 (Vaughan *et al*., 1992) to calculate the thigh center of mass given the thigh length is 0.463 m.



   Distance from hip center:
   r_{thigh} = _____ × l_{thigh} = _____ × 0.463 = _____ m

3. Use the regression equations in Table 2 (Vaughan *et al*., 1992) to calculate the thigh moments of inertia about its center of mass for a 90.0 kg person given the thigh length is 0.463 m and mid-thigh circumference is 0.445 m.

   $I_{zz}^{thigh/cm}$ = _____ × m_{total} × [(l_{thigh})^2 + _____ (c_{thigh})^2] + _____
   $I_{zz}^{thigh/cm}$ = _____ × 90.0 × [(0.463)^2 + _____ (0.445)^2] + _____ = _____ kg·m^2

   $I_{xx}^{thigh/cm}$ = _____ × m_{total} × [(l_{thigh})^2 + _____ (c_{thigh})^2] + _____
   $I_{xx}^{thigh/cm}$ = _____ × 90.0 × [(0.463)^2 + _____ (0.445)^2] + _____ = _____ kg·m^2

   $I_{yy}^{thigh/cm}$ = _____ × m_{total} × (c_{thigh})^2 + _____
   $I_{yy}^{thigh/cm}$ = _____ × 90.0 × (0.445)^2 + _____ = _____ kg·m^2

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Table 1. Body Segment Parameters by Dempster (1955)

Segment	Endpoints (proximal to distal)	Segmental mass/ total mass	Center of mass/ segment length		Radius of gyration/ segment length		
		P	$R_{proximal}$	R_{distal}	K_{cm}	$K_{proximal}$	K_{distal}
Hand	Wrist center to knuckle II of third finger	0.0060	0.506	0.494	0.298	0.587	0.577
Forearm	Elbow to wrist center	0.0160	0.430	0.570	0.303	0.526	0.647
Upper arm	Glenohumeral joint to elbow center	0.0280	0.436	0.564	0.322	0.542	0.645
Forearm and hand	Elbow to wrist center	0.0220	0.682	0.318	0.468	0.827	0.565
Upper extremity	Glenohumeral joint to wrist center	0.0500	0.530	0.470	0.368	0.645	0.596
Foot	Ankle to ball of foot	0.0145	0.500	0.500	0.475	0.690	0.690
Calf	Knee to ankle center	0.0465	0.433	0.567	0.302	0.528	0.643
Thigh	Hip to knee center	0.1000	0.433	0.567	0.323	0.540	0.653
Lower extremity	Hip to ankle center	0.1610	0.447	0.553	0.326	0.560	0.650
Head	C7-T1 to ear canal	0.0810	1.000	0.000	0.495	1.116	0.495
Trunk	Greater trochanter to glenohumeral joint	0.4970	0.495	0.505	0.406	0.640	0.648
Head, arms, and trunk	Greater trochanter to glenohumeral joint	0.6780	0.626	0.374	0.496	0.798	0.621

Dempster, W.T., 1955. *Space Requirements of the Seated Operator: Geometrical, Kinematic, and mechanical Aspects of the Body Special Reference to the Limbs*., WADC Technical Report 55-159. Wright-Patterson Air Force Base, OH.

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Table 2. Regression Equations by Vaughan *et al.* (1992)

Segment	Inertial parameter	Axis	Regression equation
Thigh	Mass		$m = 0.1032\, m_{total} + 12.76\, l_{thigh}\, c_{thigh}^2 - 1.023$
	Center of mass		$r_{proximal} = 0.39\, l_{thigh}$, $r_{distal} = 0.61\, l_{thigh}$
	Moment of inertia	Flexion/ Extension	$I_{zz}^{thigh/cm} = 0.00762\, m_{total}\left(l_{thigh}^2 + 0.076\, c_{thigh}^2\right) + 0.01153$
		Abduction/ Adduction	$I_{xx}^{thigh/cm} = 0.00726\, m_{total}\left(l_{thigh}^2 + 0.076\, c_{thigh}^2\right) + 0.01186$
		Internal/ External	$I_{yy}^{thigh/cm} = 0.00151\, m_{total}\, c_{thigh}^2 + 0.00305$
Calf	Mass		$m = 0.0226\, m_{total} + 31.33\, l_{calf}\, c_{calf}^2 - 0.016$
	Center of mass		$r_{proximal} = 0.42\, l_{calf}$, $r_{distal} = 0.58\, l_{calf}$
	Moment of inertia	Flexion/ Extension	$I_{zz}^{calf/cm} = 0.00347\, m_{total}\left(l_{calf}^2 + 0.076\, c_{calf}^2\right) + 0.00511$
		Abduction/ Adduction	$I_{xx}^{calf/cm} = 0.00387\, m_{total}\left(l_{calf}^2 + 0.076\, c_{calf}^2\right) + 0.00138$
		Internal/ External	$I_{yy}^{calf/cm} = 0.00041\, m_{total}\, c_{calf}^2 + 0.00012$
Foot	Mass		$m = 0.0083\, m_{total} + 254.5\, l_{foot}\, h_{malleolus}\, w_{malleolus} - 0.065$
	Center of mass		$r_{proximal} = 0.44\, l_{foot}$, $r_{distal} = 0.56\, l_{foot}$
	Moment of inertia	Flexion/ Extension	$I_{zz}^{foot/cm} = 0.00023\, m_{total}\left(3\, l_{foot}^2 + 4\, h_{malleolus}^2\right) + 0.00022$
		Abduction/ Adduction	$I_{xx}^{foot/cm} = 0.00021\, m_{total}\left(3\, l_{foot}^2 + 4\, w_{foot}^2\right) + 0.00067$
		Internal/ External	$I_{yy}^{foot/cm} = 0.00141\, m_{total}\left(w_{foot}^2 + h_{thigh}^2\right) - 0.00008$

l_{thigh} = thigh length (hip to knee) l_{calf} = calf length (knee to ankle) l_{foot} = foot length (heel to toe)

c_{thigh} = mid-thigh circumference c_{calf} = calf circumference $h_{malleolus}$ = lateral malleolus height

m_{total} = total body mass w_{foot} = foot width (breadth of metatarsals) $w_{malleolus}$ = malleolus width (medial to lateral)

Vaughan, C.L., Davis, B.L., O'Connor, J.C., 1992. *Dynamics of Human Gait*. Champaign, IL: Human Kinetics.

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