Modeling Forces Generated by Muscles and Tendons

BioE215 Physics-based Simulation of Biological Structures

May 11, 2007

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Paul and Jeff may tell you differently, but…
without forces, mass is irrelevant ! ! !

$$\vec{F} = m\vec{a}$$

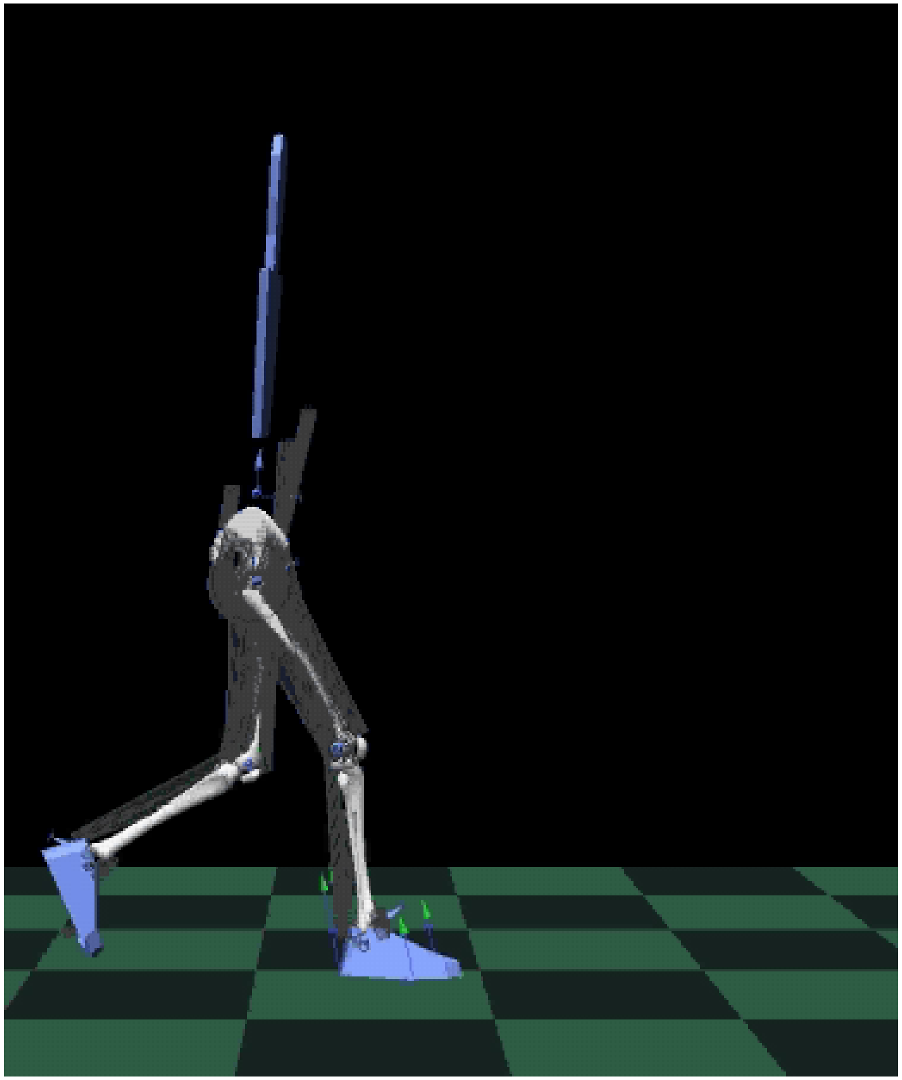
$$\vec{a} = \frac{\vec{F}}{m} = \frac{\vec{0}}{m} = \vec{0}$$

Why are muscle forces important?

- Moving (walking, running, waving, …)
- Talking
- Breathing
- Seeing
- Hearing
- Digesting (smooth muscle)
- Pumping (cardiac muscle)

We need muscles to move

Without muscles…



Muscle & tendon properties influence performance

Muscle strength and the rate at which muscles contract are major determinants of running speed.

Kangaroos can run more efficiently at fast speeds than at slow speeds, partly because of the compliance of their tendons.

Joint contact forces are largely due to muscle forces

Strong quadriceps can lift a small car off the ground.

$$F_o^M \approx 10,000\,N \approx 1,000\ kg$$

$$VW\ \ Bug \approx 1,180\ kg.$$

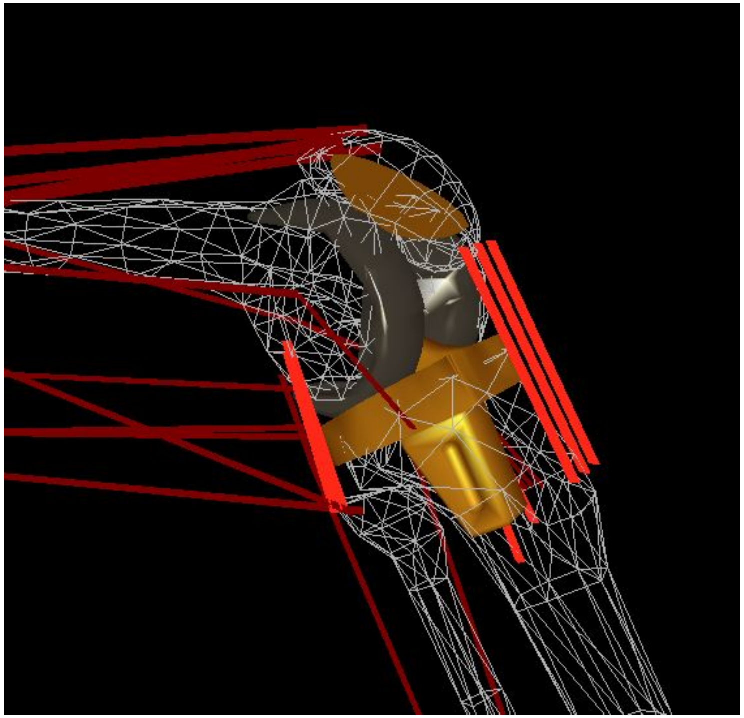
Joint contact forces:

3 * Body Weight during walking

5 * Body Weight during running

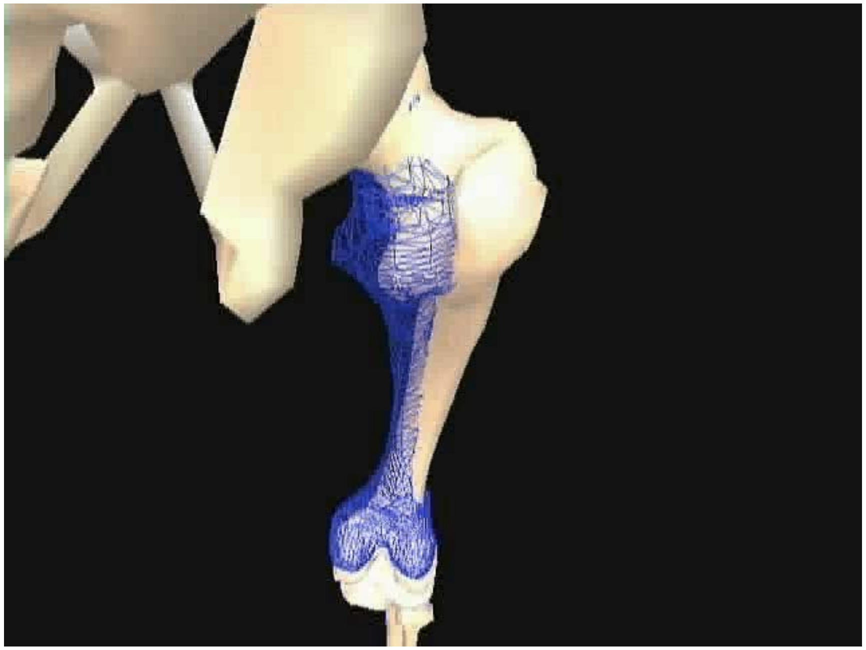
Joint contact forces are largely due to muscle forces

Joint Disease and Joint Replacements



Piazza & Delp, 2001

Bone Development

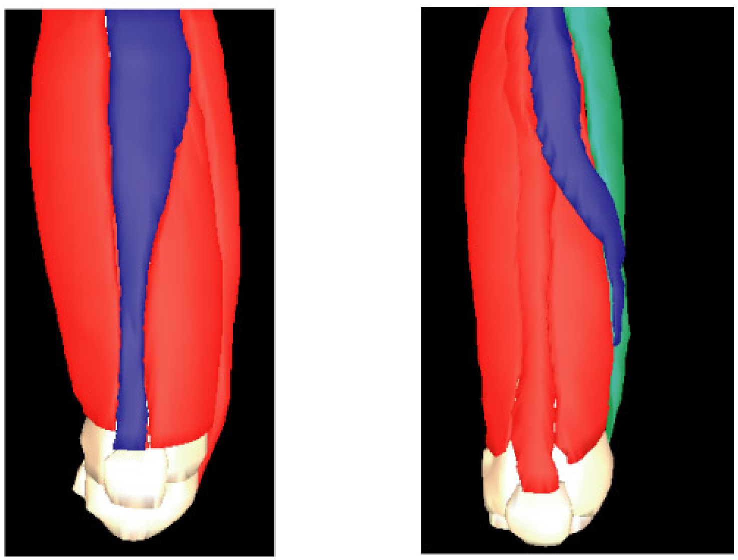


Arnold & Delp, 2001

Osteoarthritis, Osteoporosis, Bone loss in space

Muscles are the targets of treatments

Non-Operated Transferred



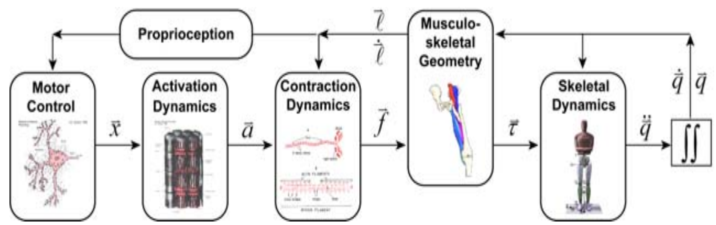
Rectus Femoris

Stiff-Knee Gait

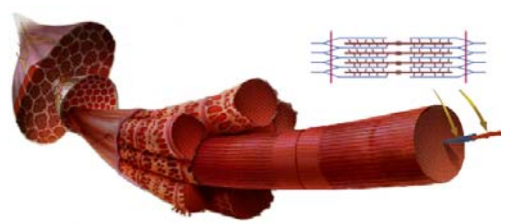
Connecticut Children's Medical Center

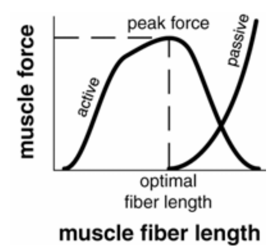
What we'll cover today

- Muscles & simulation

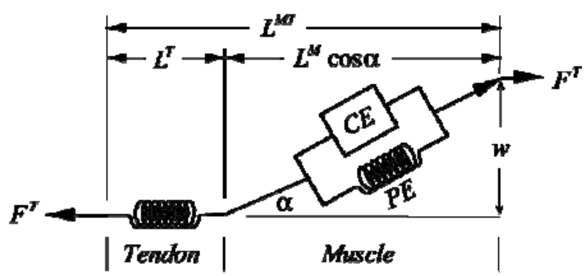


- Muscle mechanics (biology)





- Muscle models



$$F^T = f(a, F_o^M, L^{MT}, \dot{L}^{MT})$$

$$\frac{dF^T}{dt} = f(a, F_o^M, L^{MT}, \dot{L}^{MT})$$

What we'll cover today

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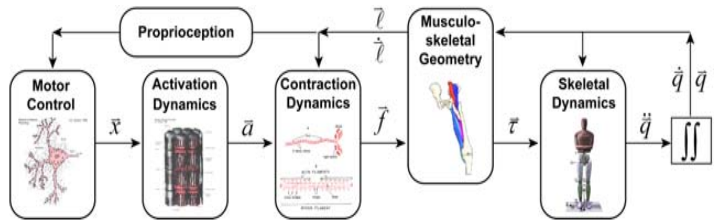

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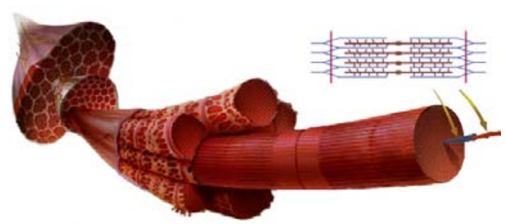

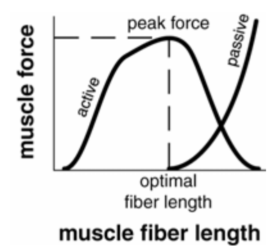

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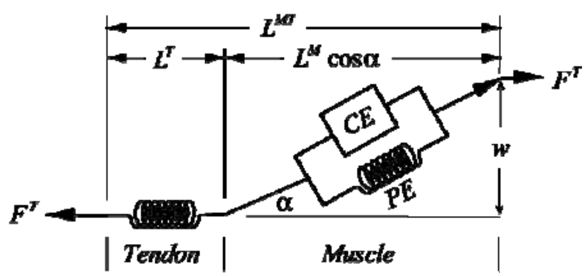

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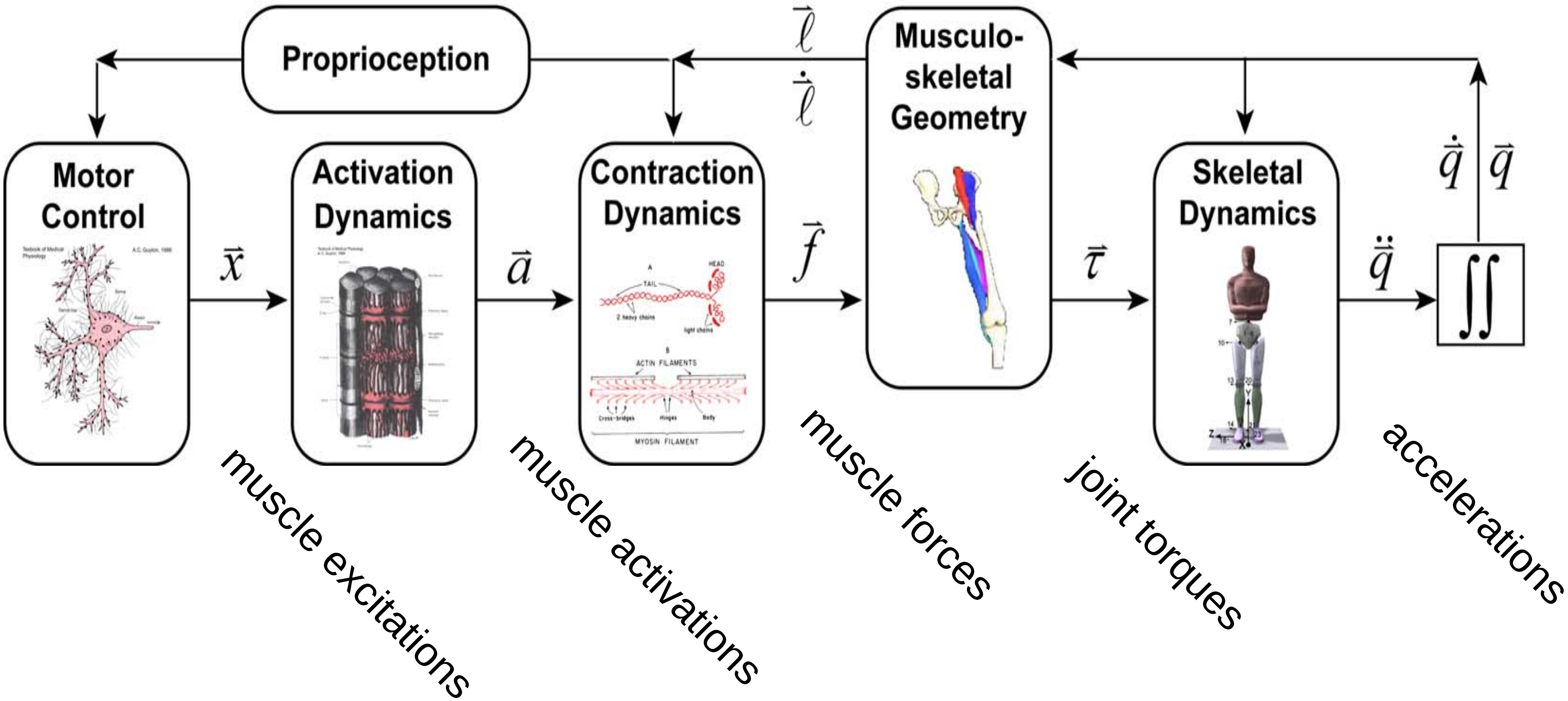
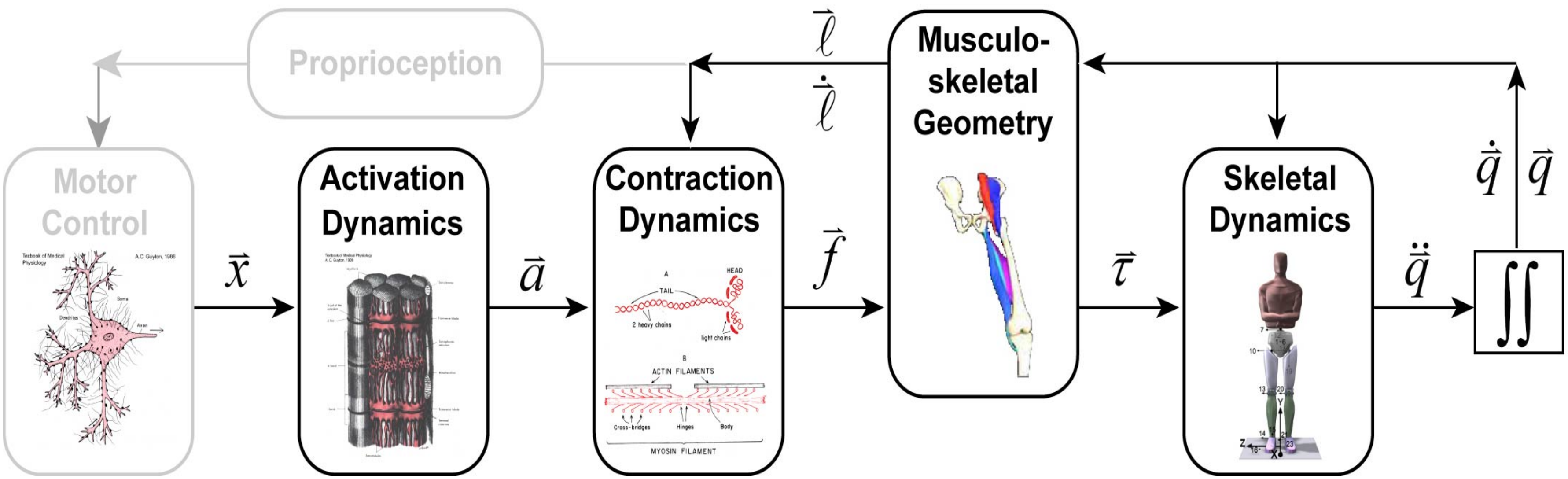
How movement is generated



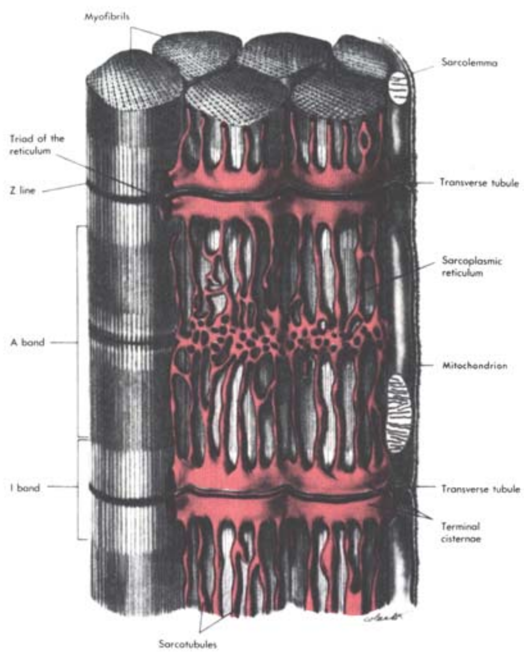
Simulation



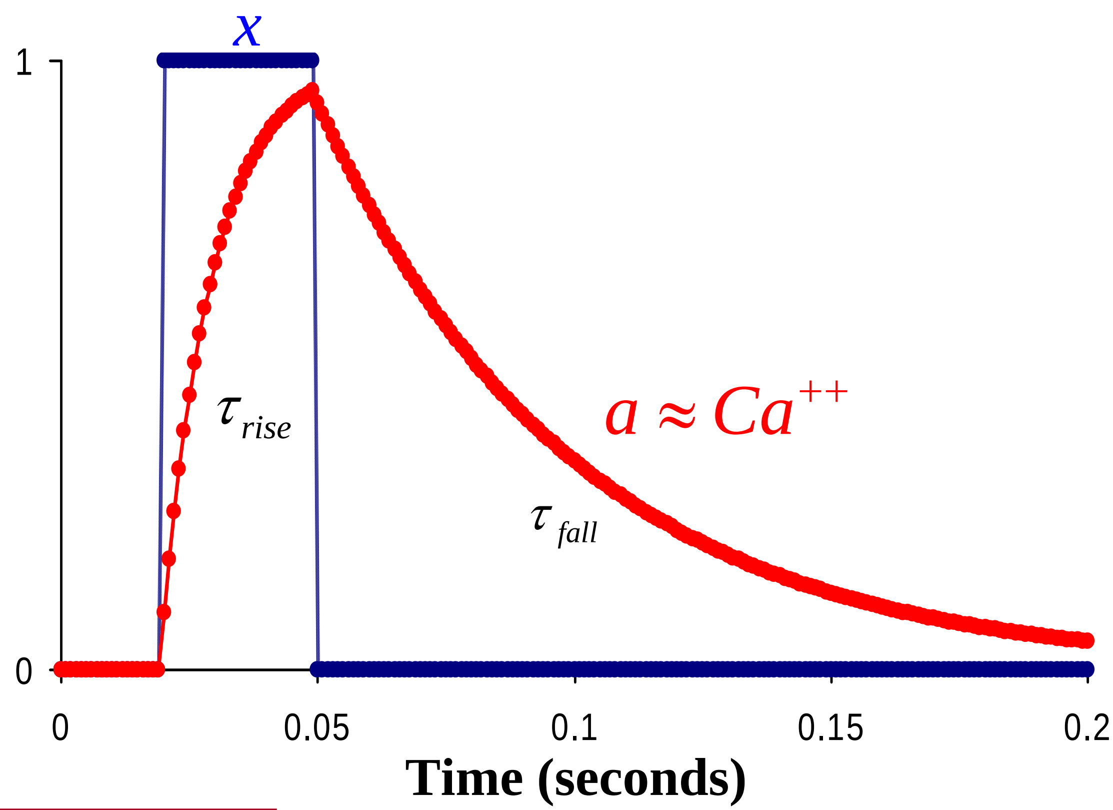
Proprioception
Motor Control
Activation Dynamics
Contraction Dynamics
Musculo-skeletal Geometry
Skeletal Dynamics
$\vec{x}$
$\vec{a}$
$\vec{f}$
$\vec{\tau}$
$\ddot{\vec{q}}$
$\dot{\vec{q}}$
$\vec{q}$
$\vec{\ell}$
$\dot{\vec{\ell}}$
$\iint$

Activation dynamics

Textbook of Medical Physiology
A. C. Guyton, 1986

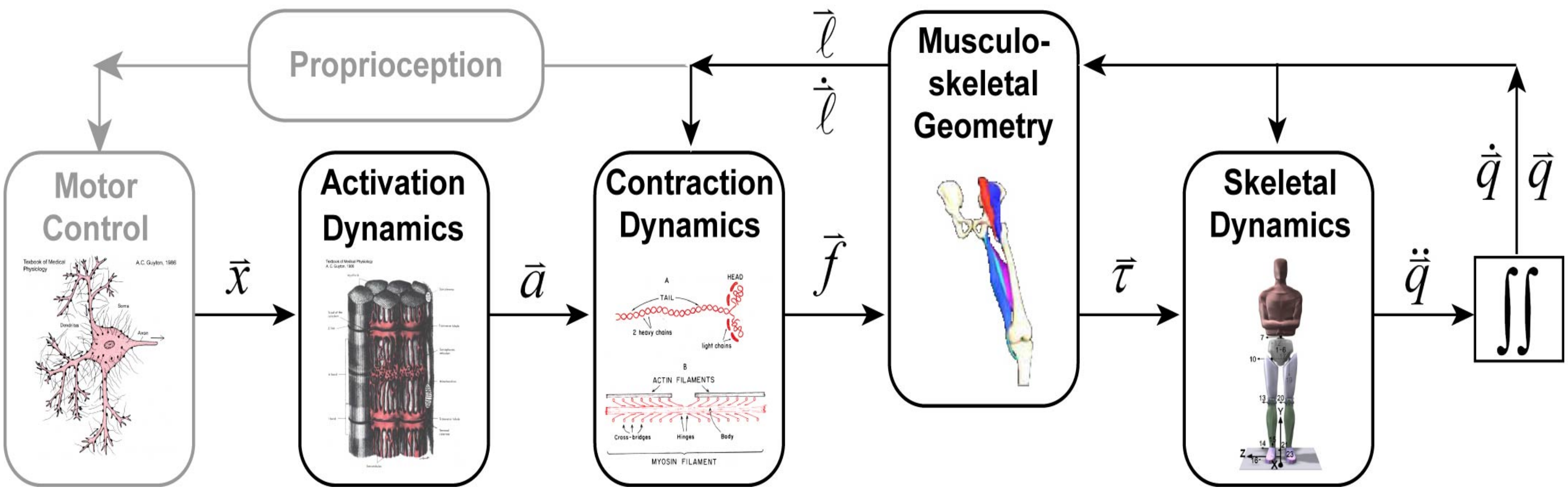


Excitation-Contraction Coupling

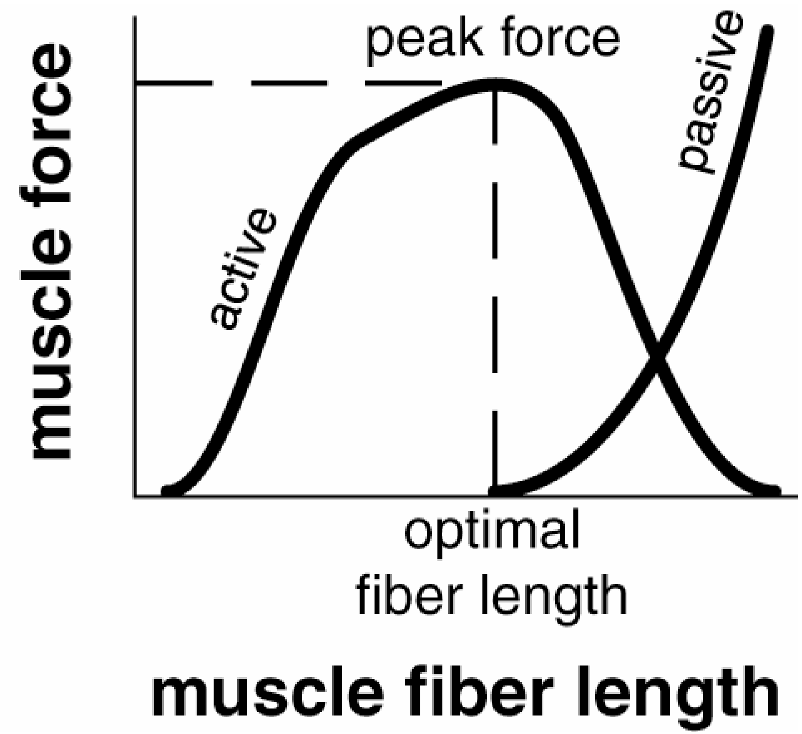


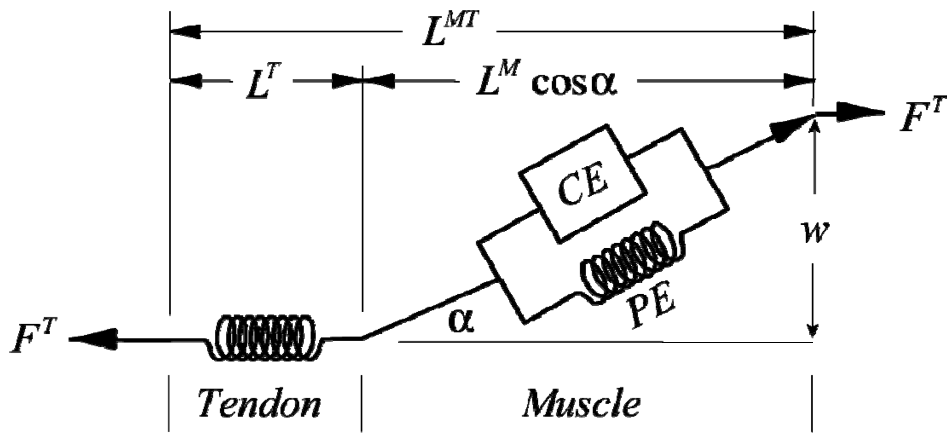
$$\frac{da}{dt} = \frac{x^2 - xa}{\tau_{rise}} + \frac{x - a}{\tau_{fall}}$$

Contraction dynamics



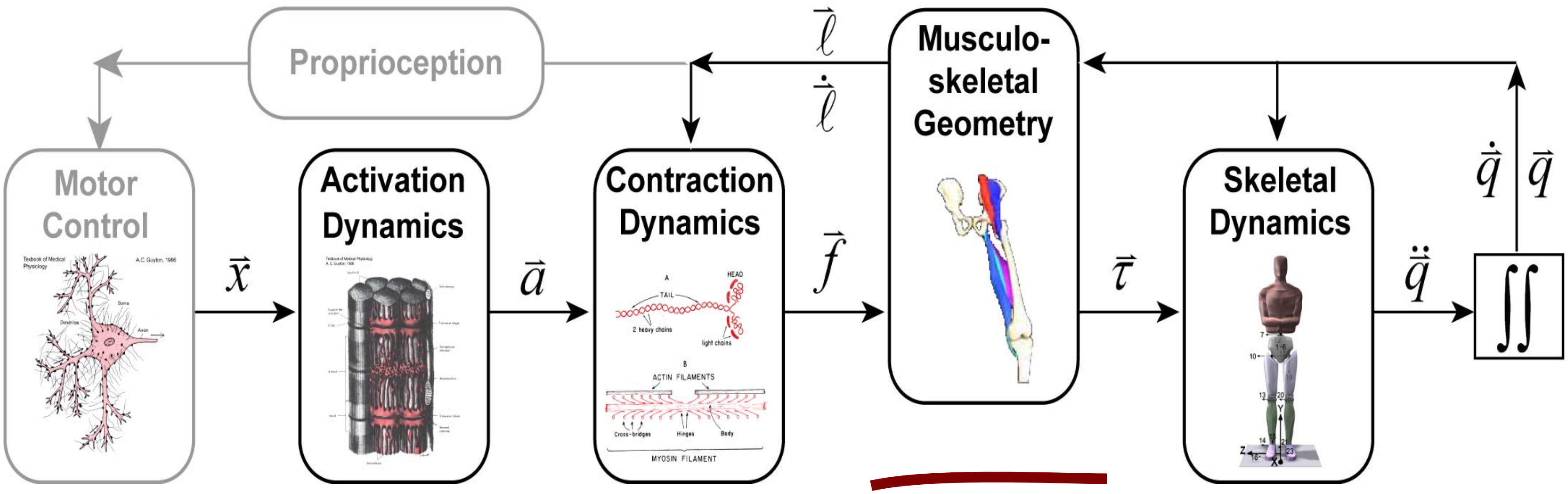
Contraction dynamics (Muscle Mechanics)



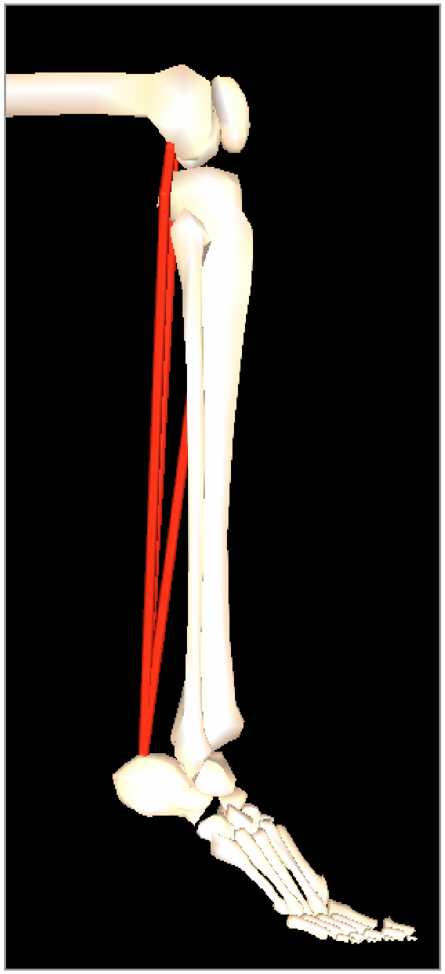


$$\frac{dF^T}{dt} = f(a, F^T, L^{MT}, \dot{L}^{MT})$$

Forward Dynamic Simulation

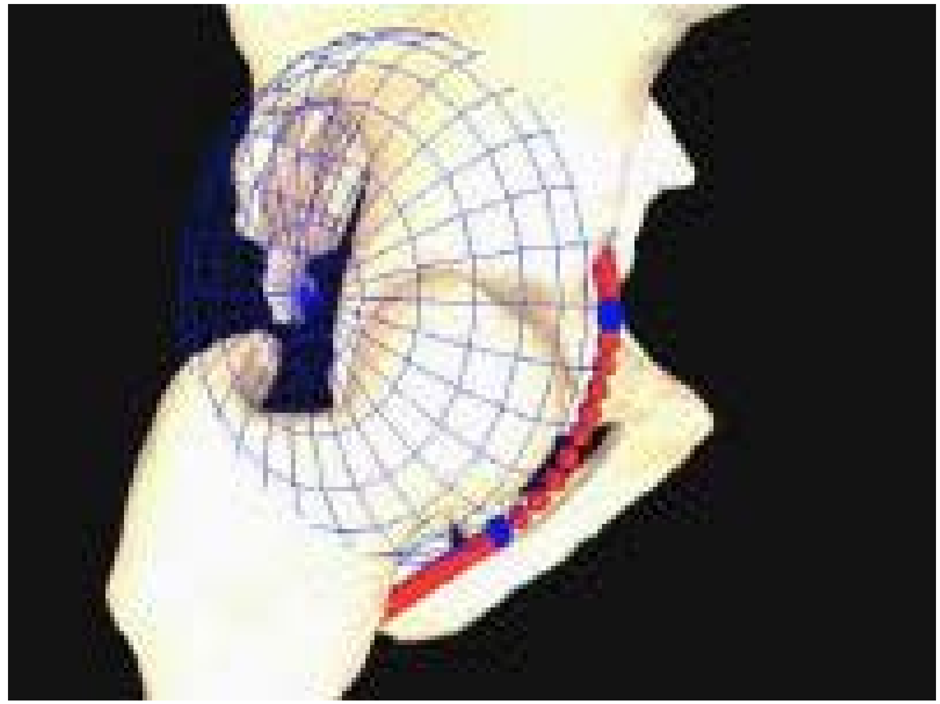


Musculoskeletal Geometry (SIMM)



Straight Lines

Wrapping



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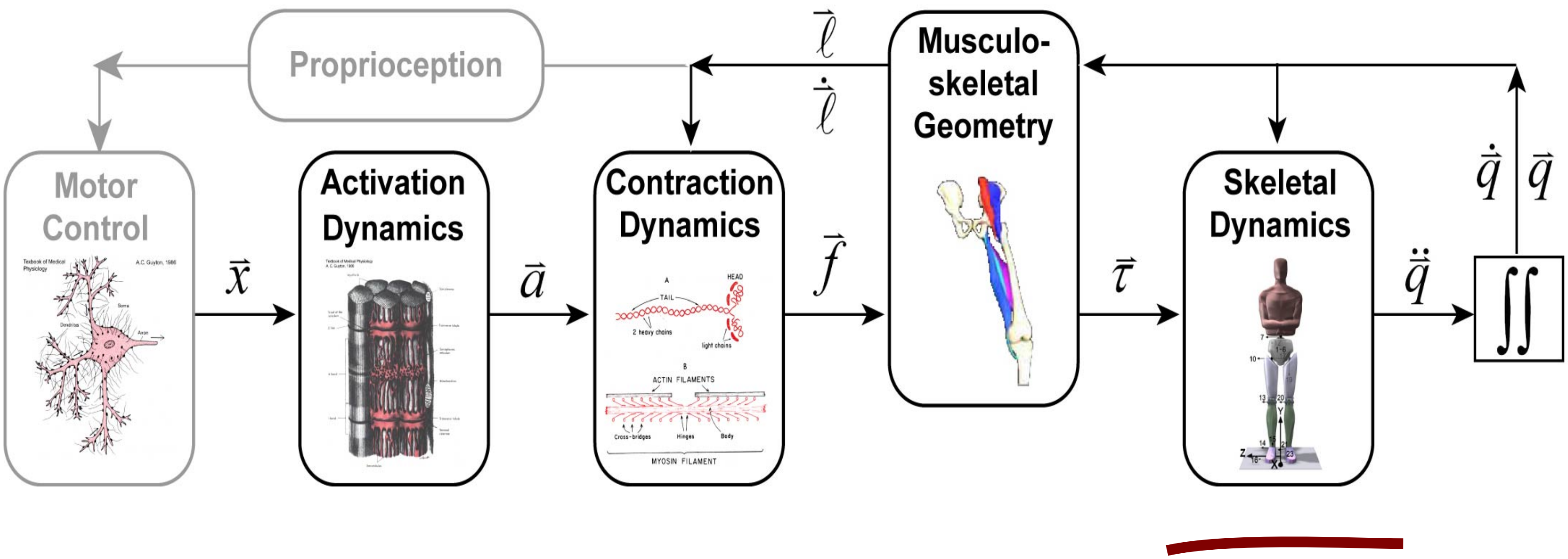
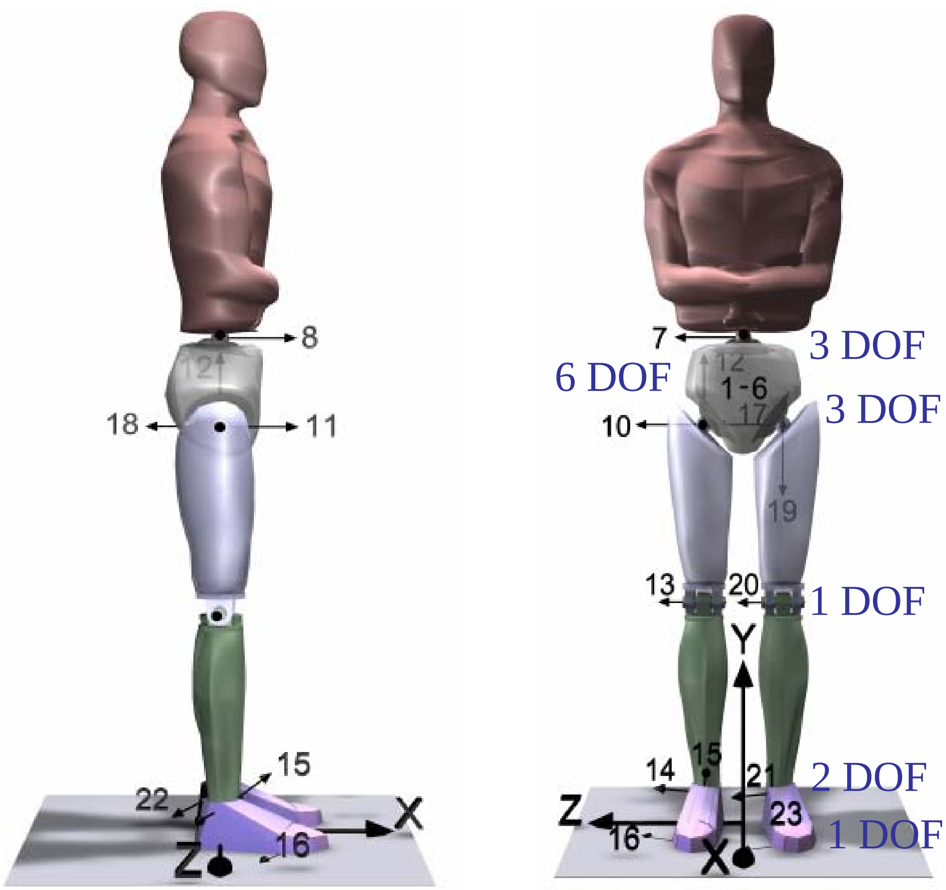


Skeletal Dynamics

10 Rigid Bodies
- mass
- center of mass
- inertia tensor

10 Joints

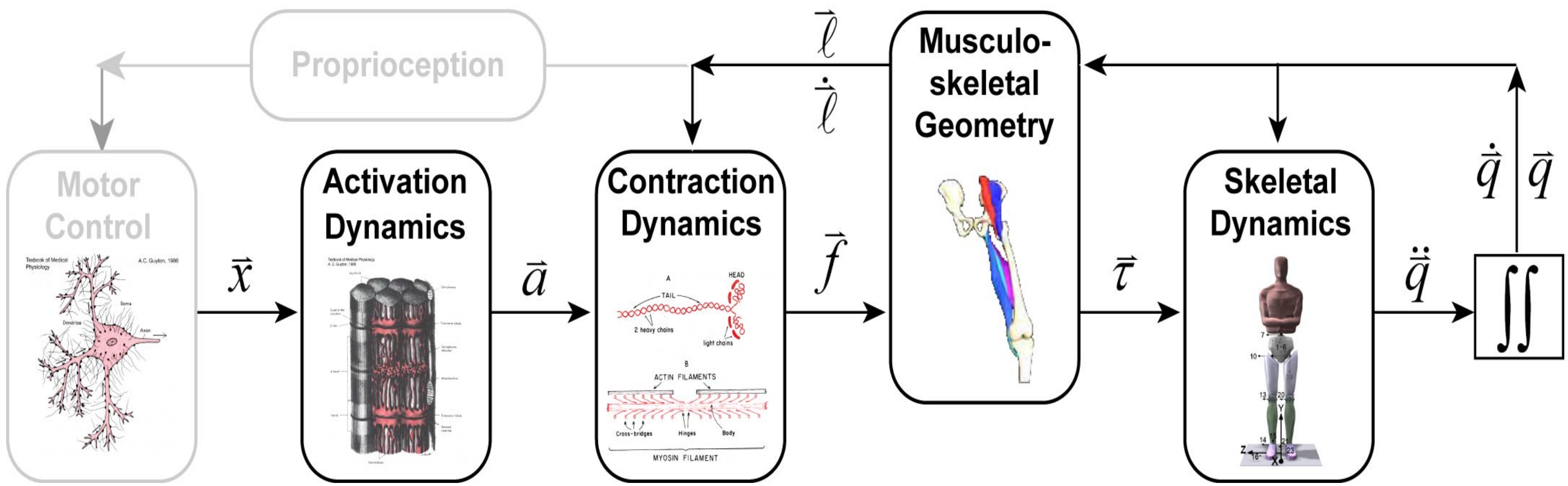
23 Degrees of Freedom



Equations of Motion

$$\frac{d^2 \vec{q}}{dt^2} = \overleftrightarrow{I}^{-1} \cdot \left\{ \vec{G} + \vec{C} + \overleftrightarrow{R} \cdot \vec{f}_{mt} + \vec{F}_{ext} \right\}$$

Simulating the Musculoskeletal System

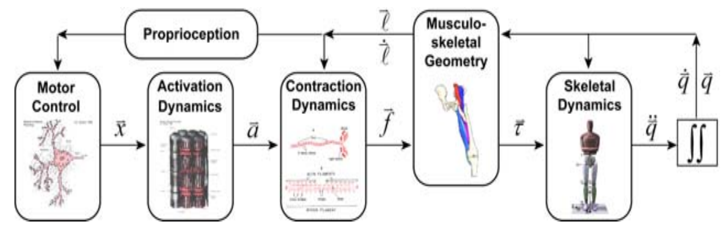


Muscle/Tendon Mechanics
(How muscles and tendons generate force)

Simbody

What we'll cover today

- Muscles & simulation



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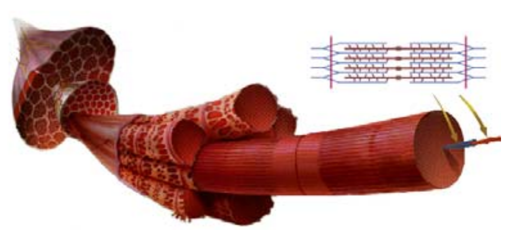
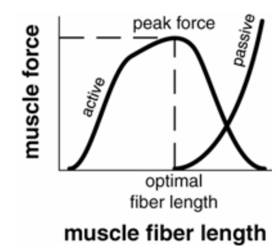
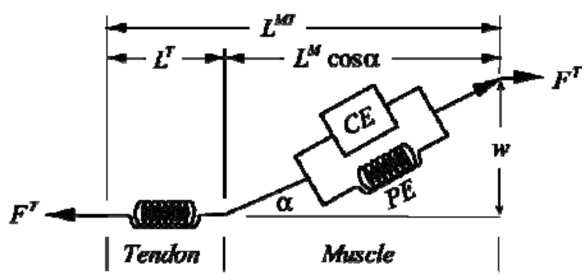




- Muscle models



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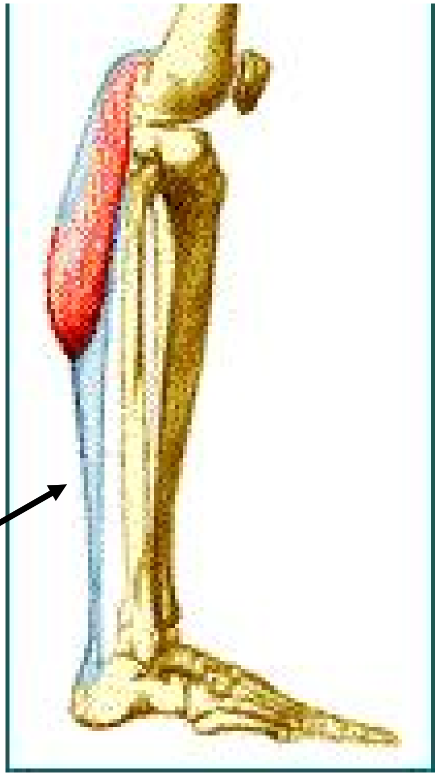
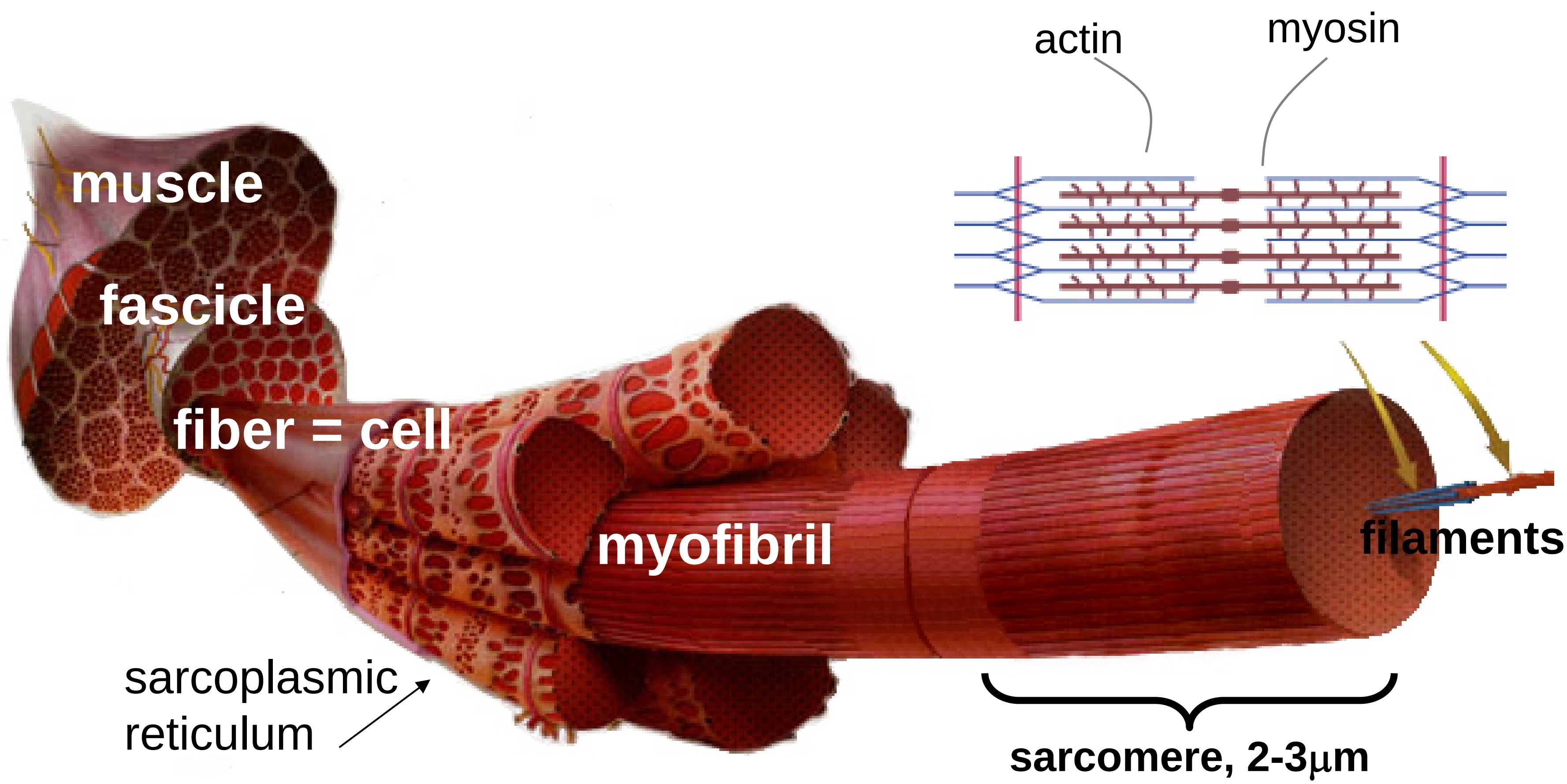
Muscles actuate movement by developing tension

- Muscles pull, not push.
- Muscles are grouped into antagonistic pairs.
- Tendon connects muscle to bone.

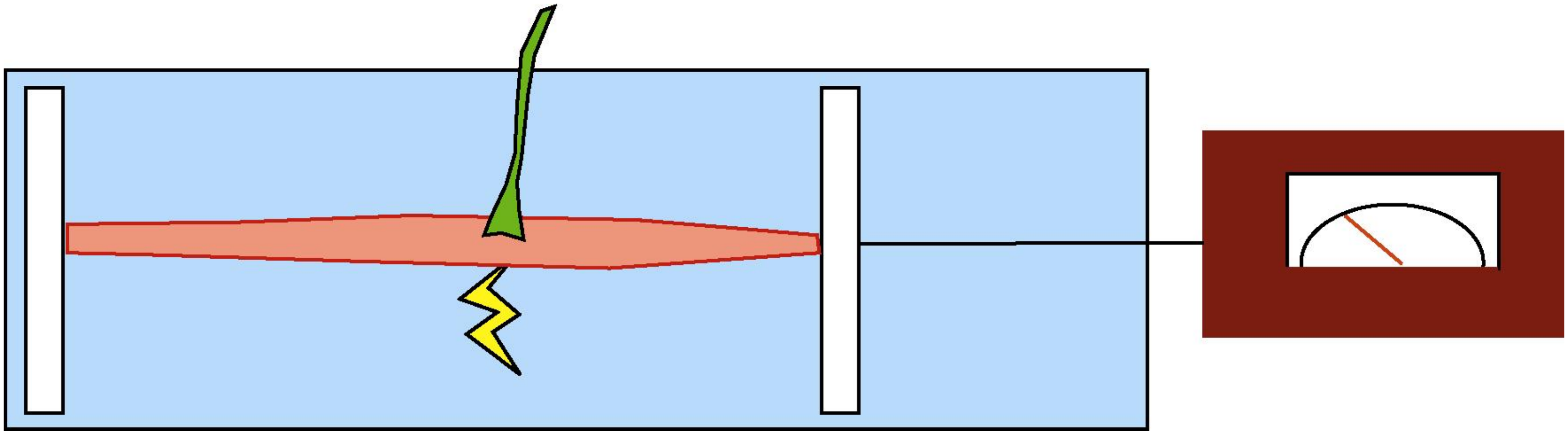


Hierarchical Muscle Structure



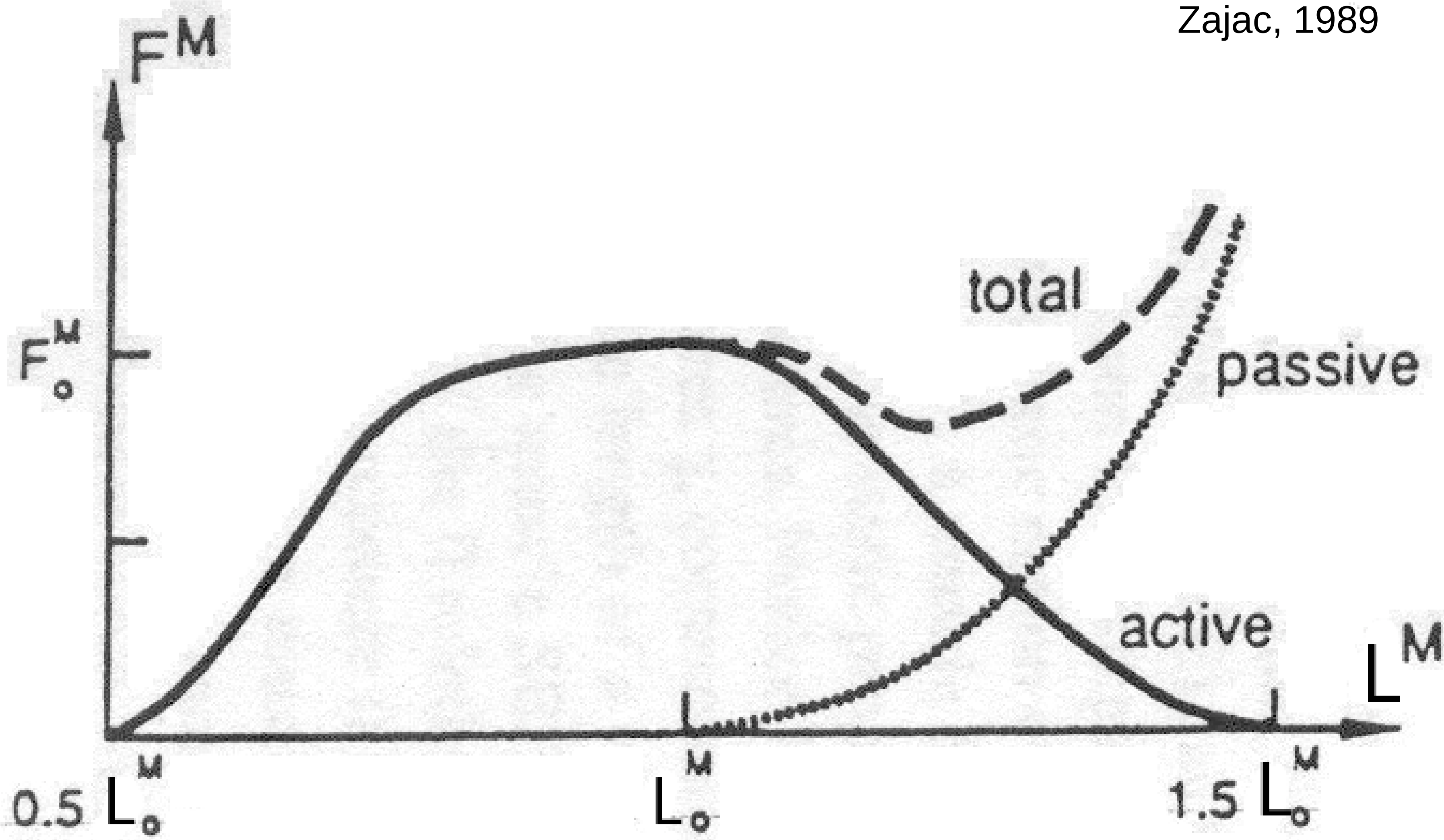
Adapted from Scientific American, September 2000

Measuring the force-length property of muscle



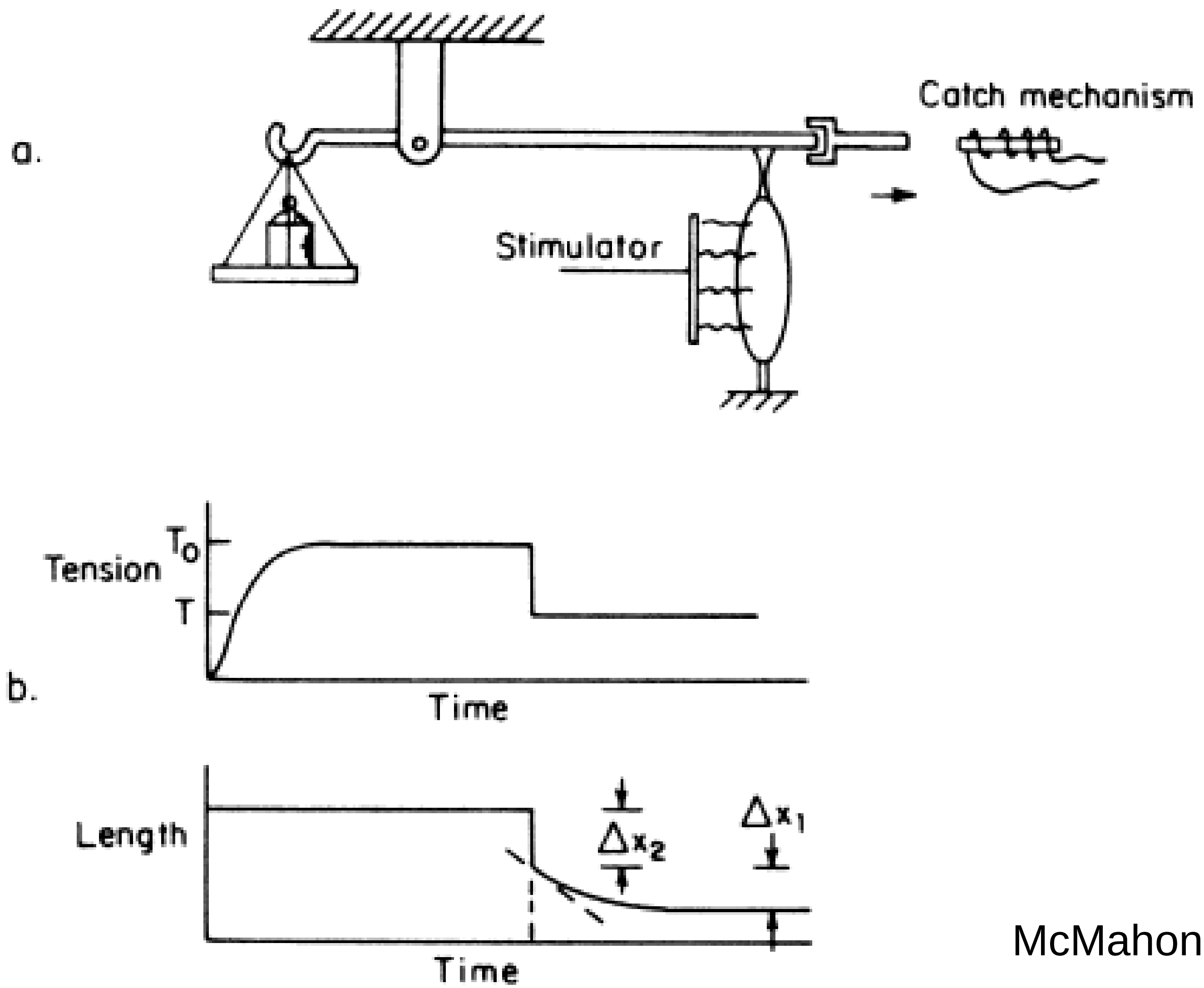
Force-length property of muscle

Zajac, 1989



Muscle Fiber Length

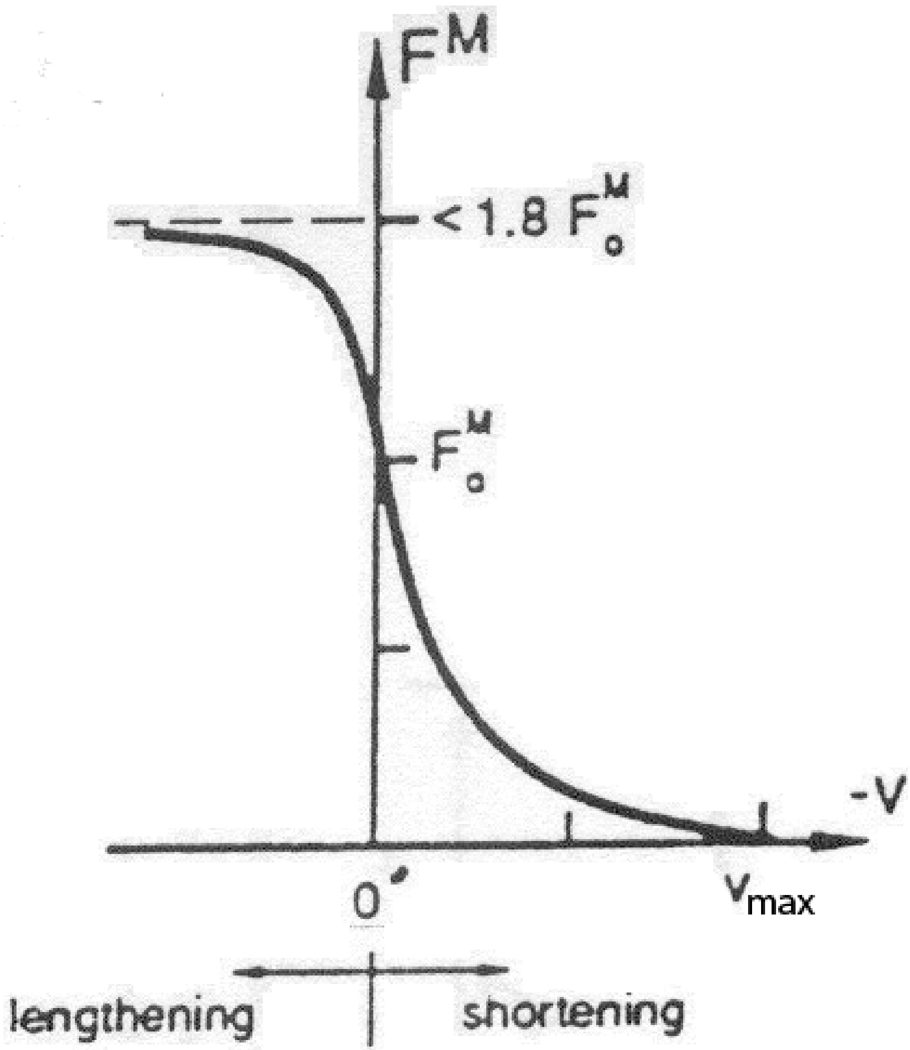
Measuring the force-velocity property of muscle



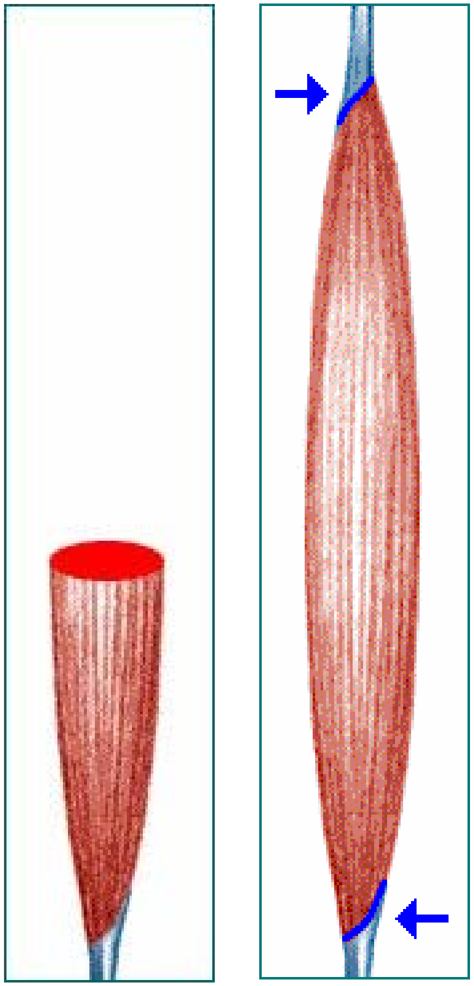
McMahon, 1984

Force-velocity property

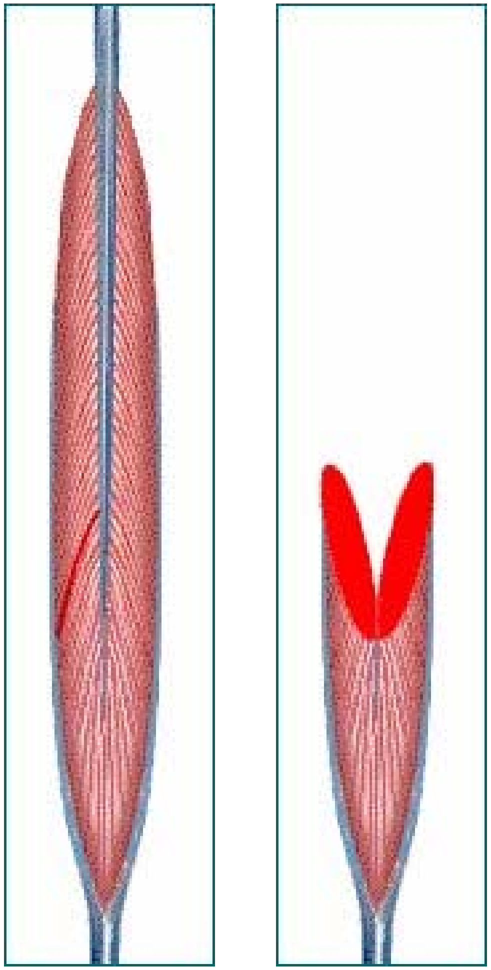
Zajac, 1989



Muscle architecture



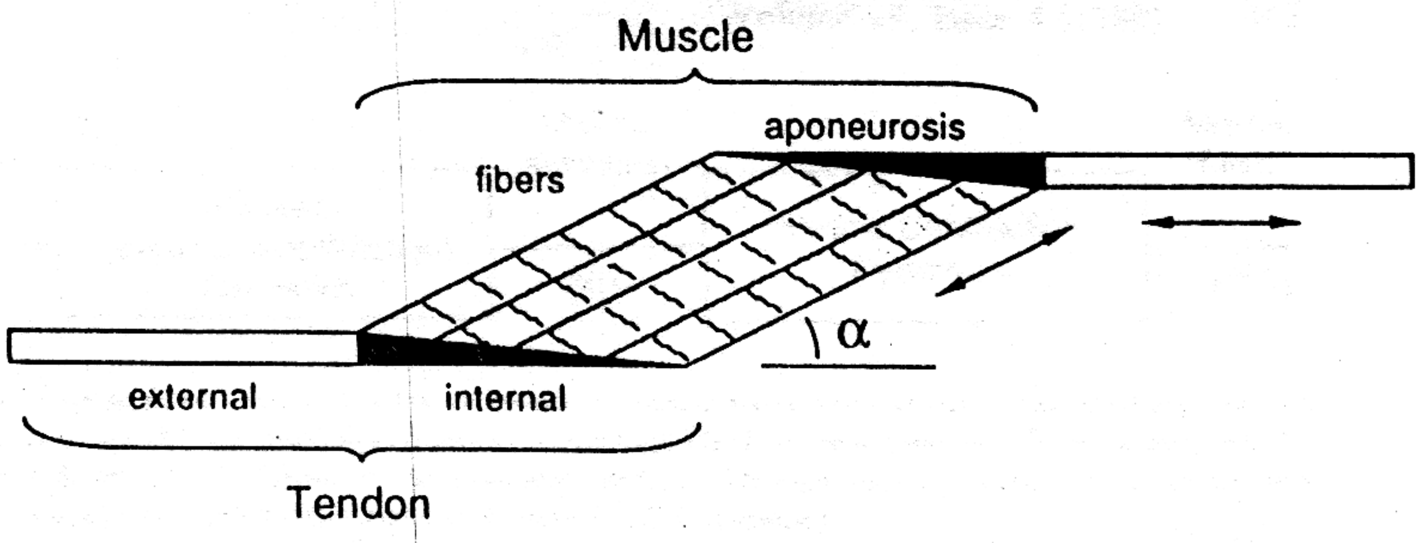
Parallel Fibered

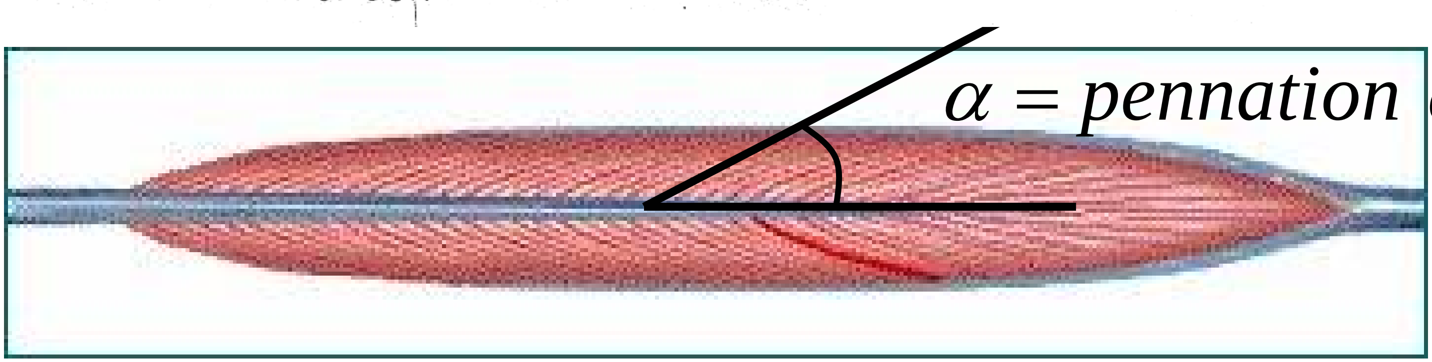


Pennate Muscle

Muscle, tendon, & pennation angle

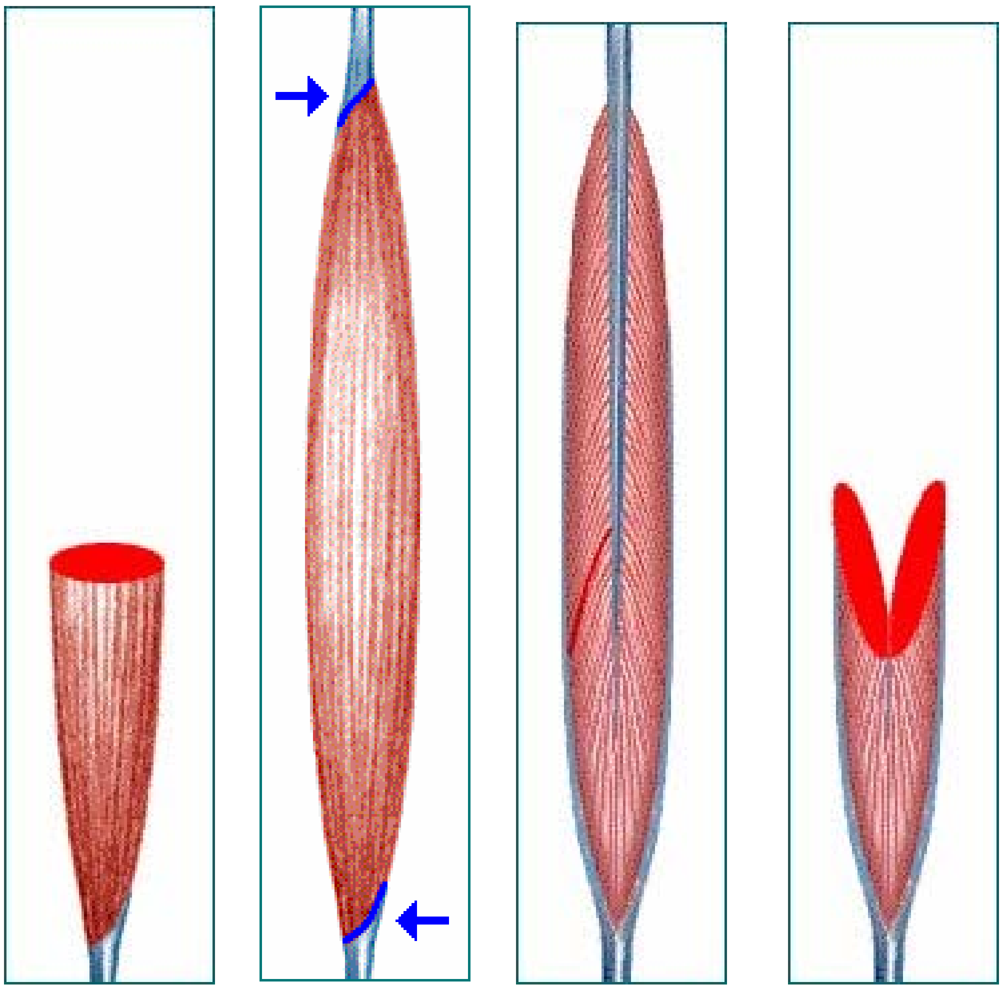
Zajac, 1989





$$\alpha = pennation\ angle$$

Muscle architecture



Parallel Fibered   Pennate Muscle

$$F_o^M \propto \; no. \; fibers$$

$$F_o^M \propto \frac{Volume}{L_o^M} \equiv PCSA$$

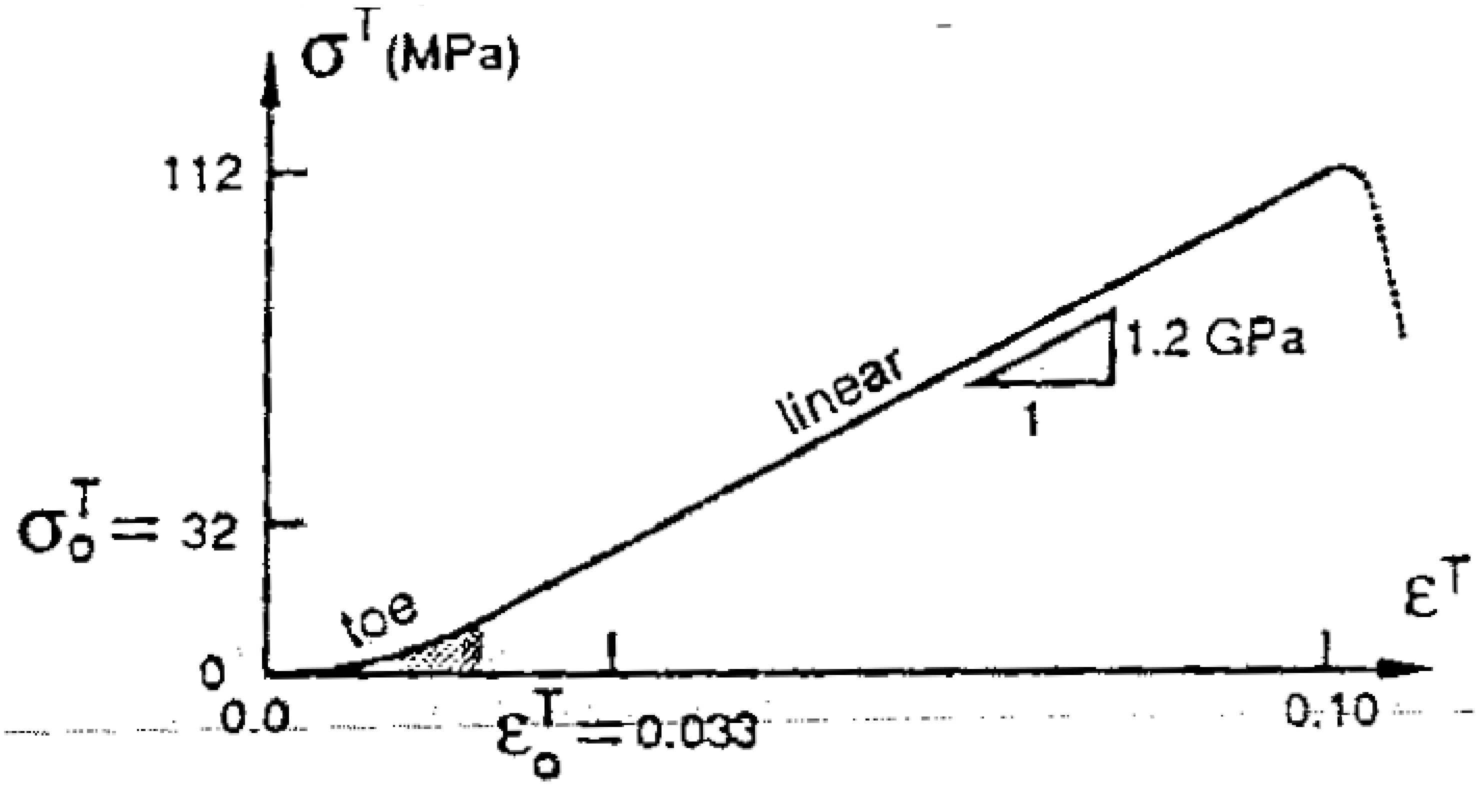
Physiologic Cross-Sectional Area

$$F_o^M = \sigma^M \cdot PCSA$$

$$\sigma^M \approx 33 \; N / \; cm^2$$

Tendon stress-strain properties

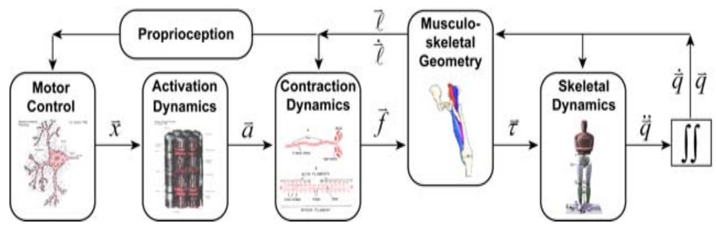
Zajac, 1989



$$\varepsilon^T = \frac{L^T - L_s^T}{L_s^T}$$

What we'll cover today

- Muscles & simulation



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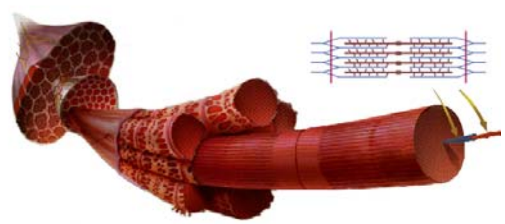
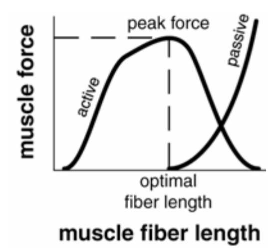
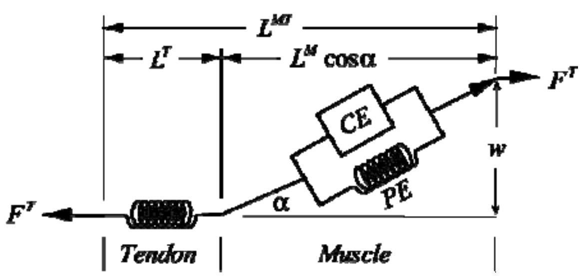




- Muscle models



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Classification of muscle models

"The formulation of a satisfactory quantitative representation of contraction dynamics has been elusive." Zahalak, 1992

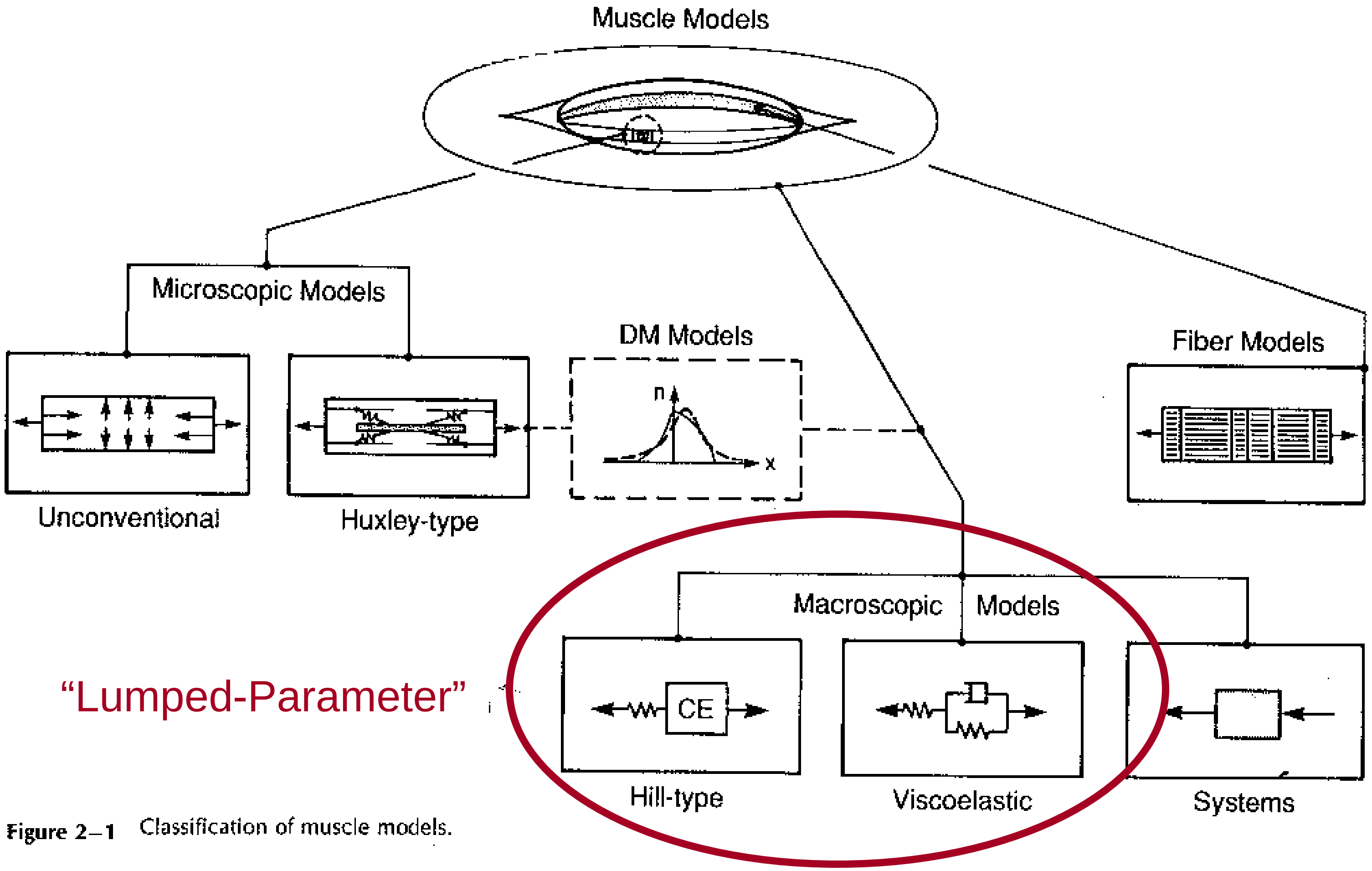


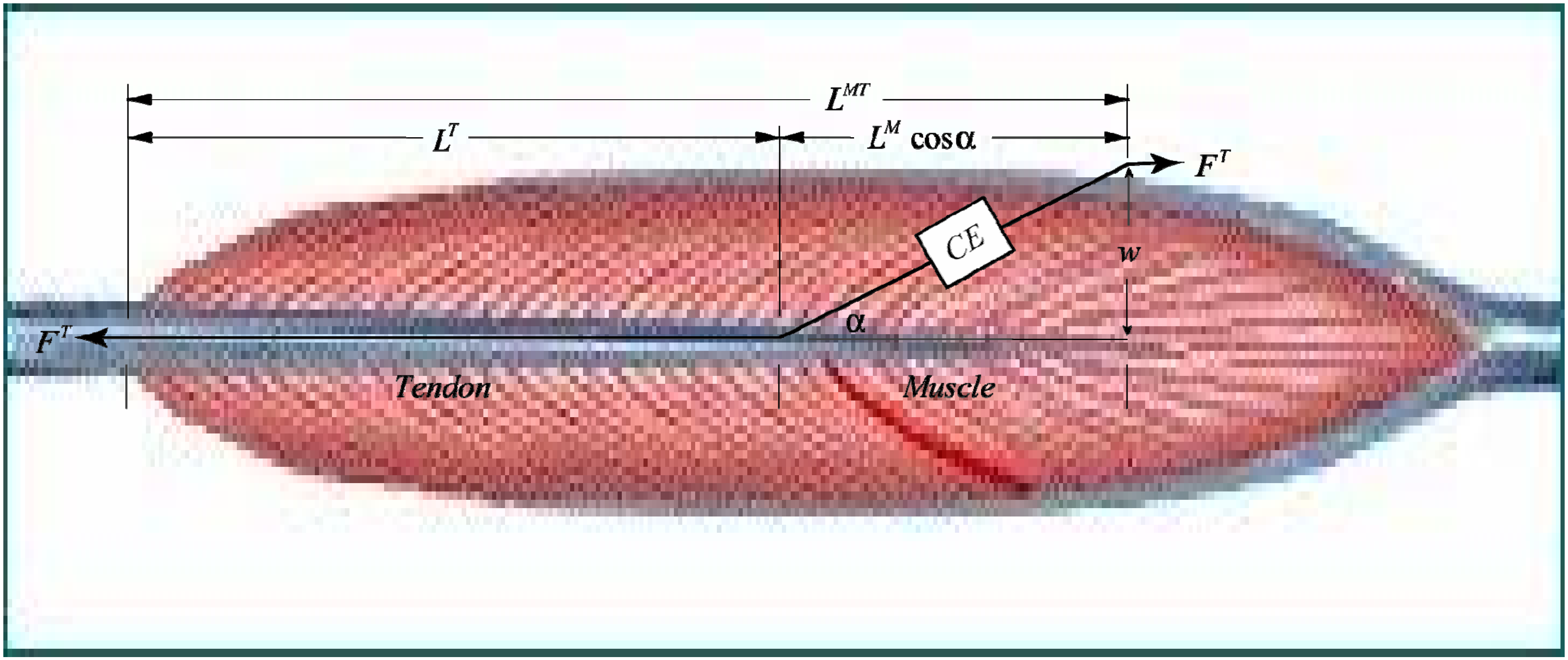
Figure 2–1 Classification of muscle models.

Some concepts

Lumped-Parameter Model

The distributed properties of all muscle fibers are lumped into a single ideal fiber characterized by parameters appropriate for the whole muscle.

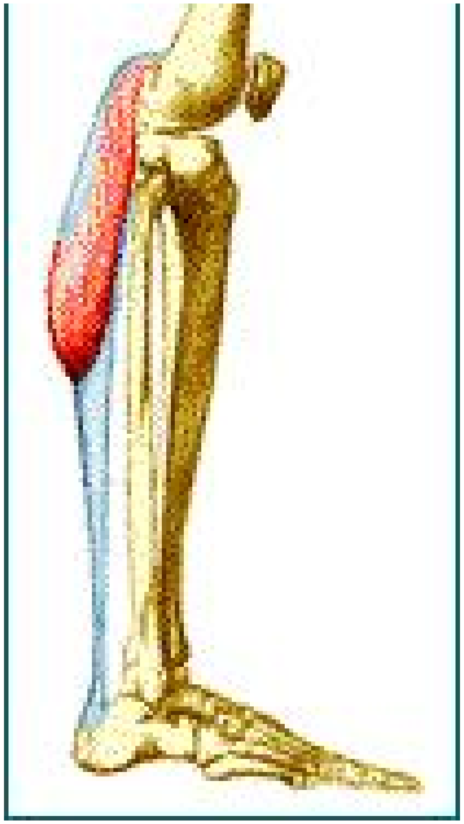
- All fibers are the same length, at the same pennation angle, etc.
- The strength of the muscle is the summed strength of the individual fibers.

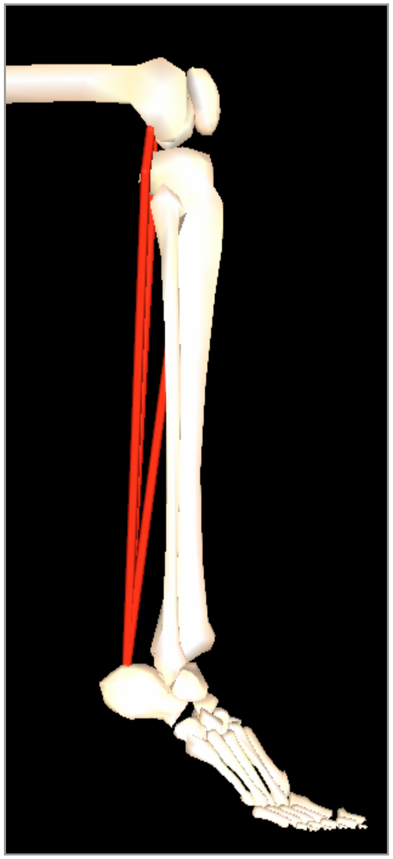


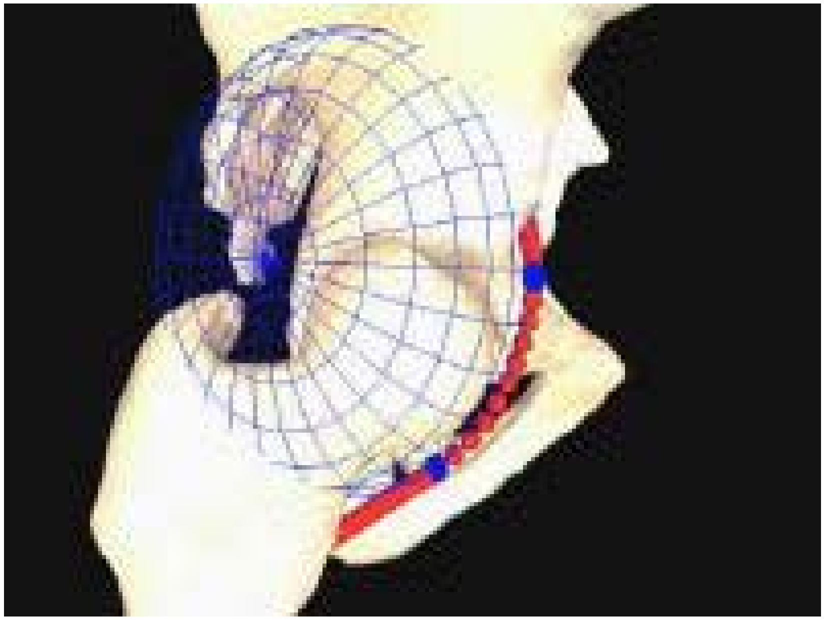
Some concepts

Line of Action

Forces act along a line connecting the muscle origin and insertion.





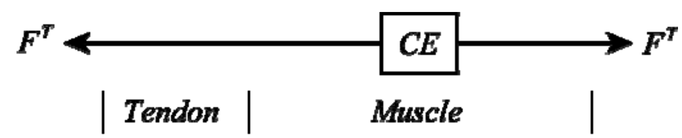


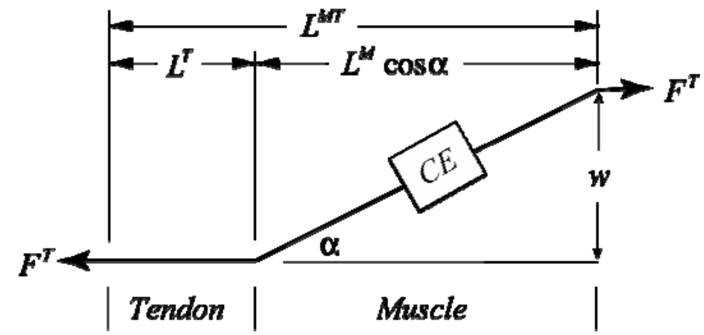
Models in the handout

a. Parallel Fibered

b. Pennate

1.





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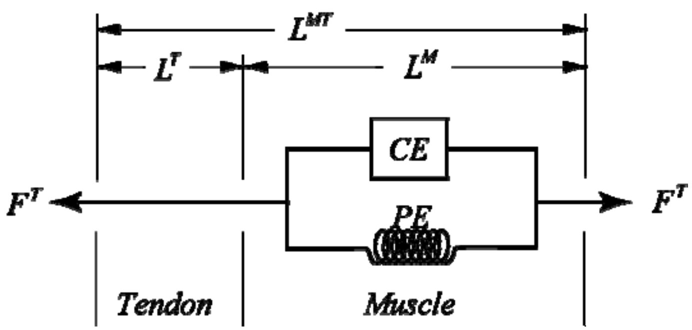
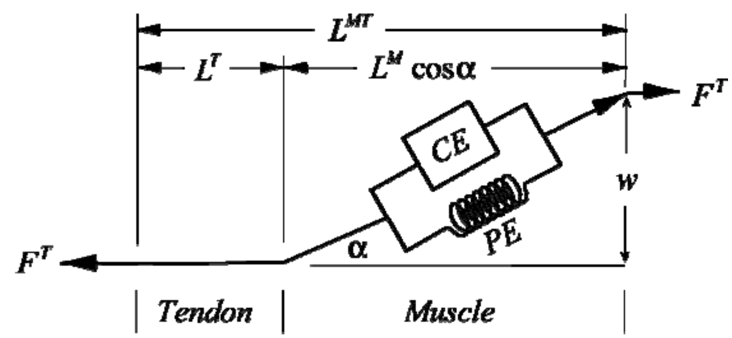
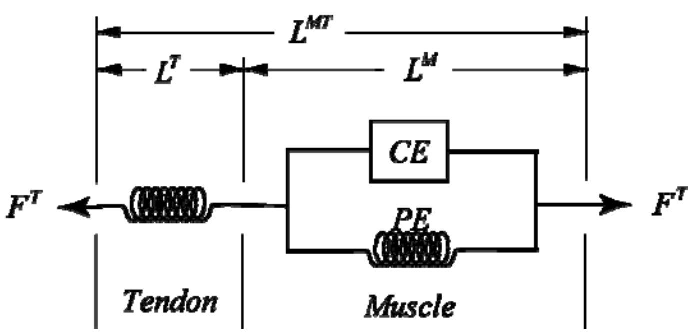
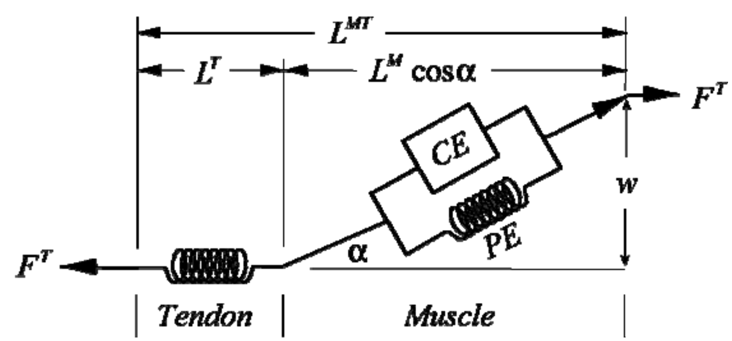


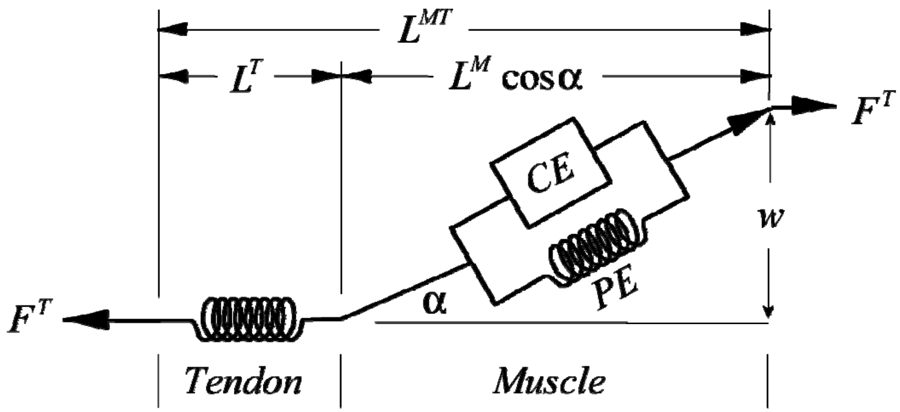


3.





Some definitions



L = Length
F = Force

T = Tendon
M = Muscle
α = Pennation angle

L^T = Length of tendon
L^M = Length of muscle
L^{MT} = Length of actuator

CE = Contractile Element.
Models the active force generating properties of muscle.

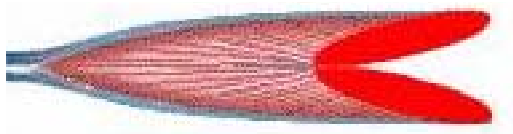
PE = Parallel Elastic Element
Models the passive force generating properties of muscle.

Five parameters you need to know

F_o^M Optimal muscle force.
Maximum isometric strength of muscle.

$$F_o^M = \sigma^M \cdot \frac{Volume}{L_o^M}$$

L_o^M Optimal muscle fiber length.
Length at which F_o^M is generated.



α_o Optimal pennation angle.
Pennation angle when the fibers are at L_o^M.

$\tilde{V}_{max}^M$ Maximum shortening velocity of muscle normalized by fiber length.

$$\underbrace{2.0 \cdot L_o^M}_{slow\ twitch} < V_{max}^M < \underbrace{10 \cdot L_o^M}_{fast\ twitch}$$

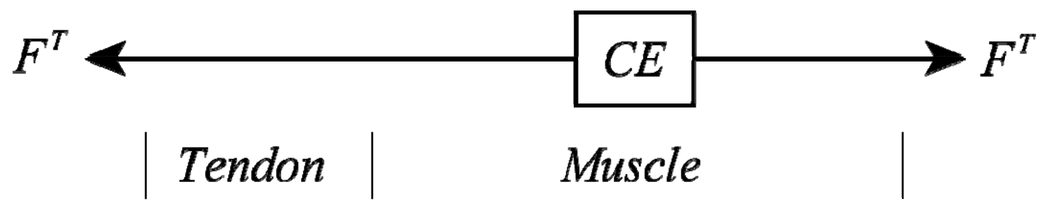
L_s^T Slack length of tendon.
Length at which tendon starts to develop force.

$$L_s^T = L_{external}^T + L_{internal}^T$$

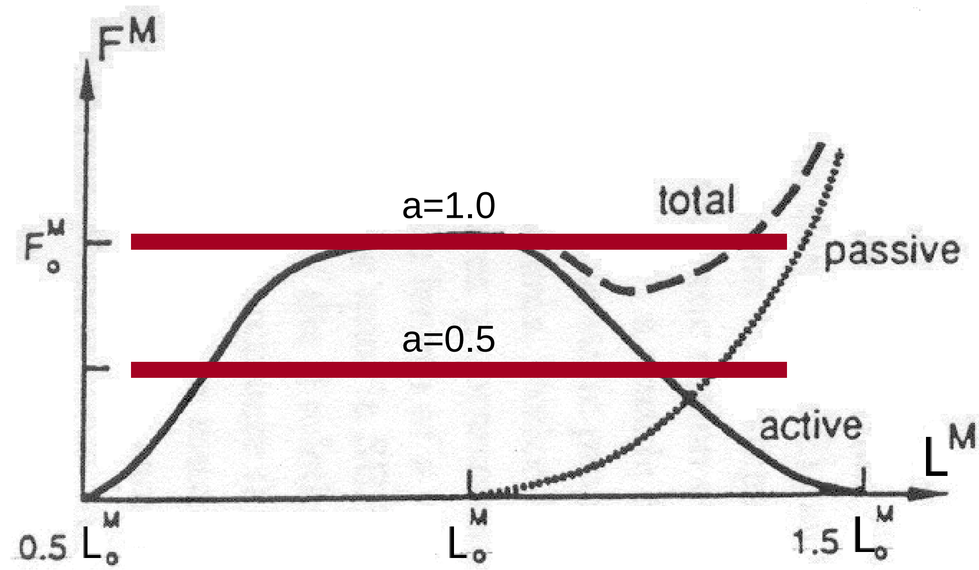
1a. Simplest

- Assumptions
  - Tendon is inelastic
  - No dependence on length or velocity
  - Parallel fibered
- Parameters
  F_o^M
- Time-varying inputs
  a

$$0\ (off) \leq a \leq 1.0\ (fully\ on)$$



$$F^T = F^{CE} = a(t) \cdot F_o^M$$

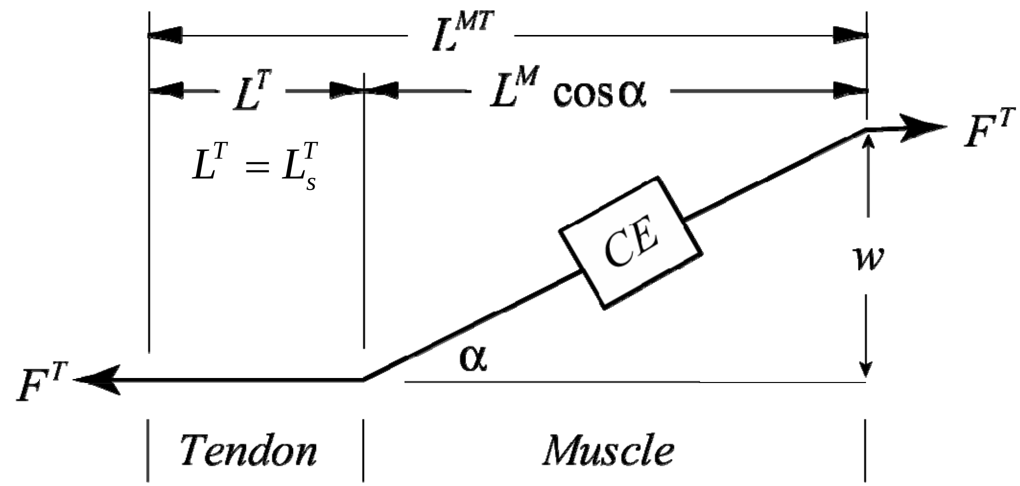


1b. Simplest with pennate fibers

- Assumptions
  - Tendon is inelastic
  - No dependence on length or velocity
  - Pennate
- Parameters

  $F_o^M \quad L_o^M \quad \alpha_o \quad L_s^T$
- Time-varying inputs

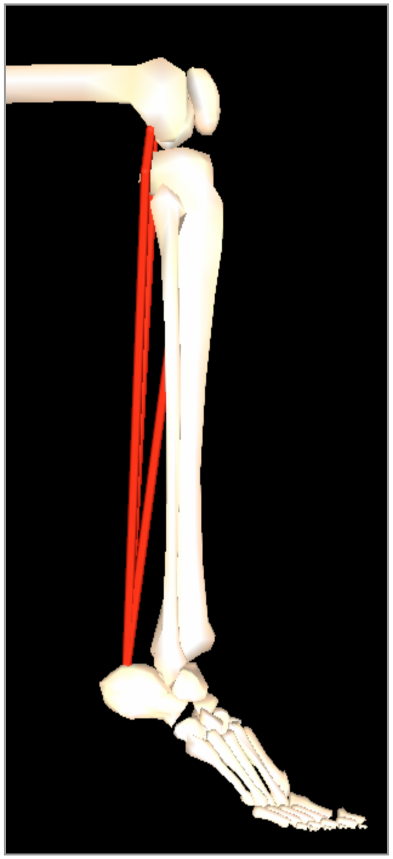
  $a \quad L^{MT}$

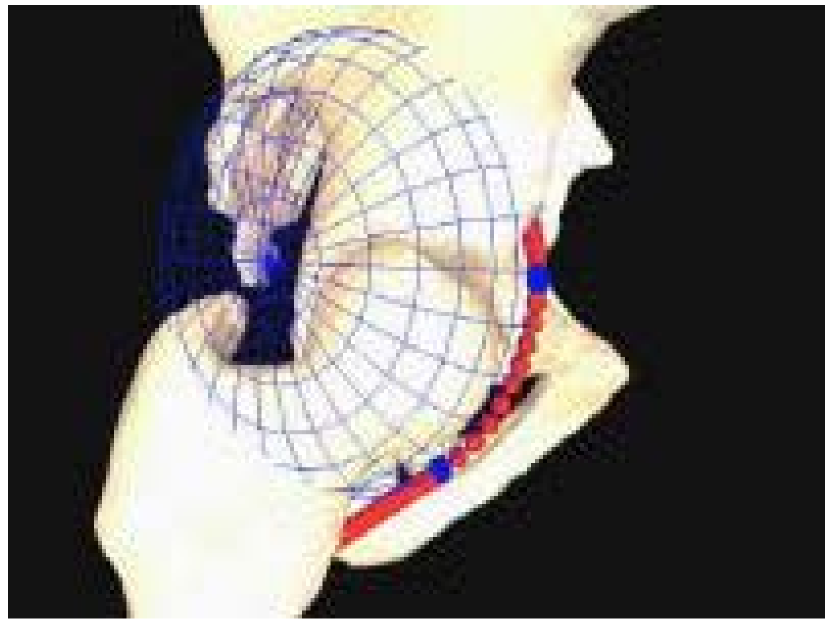


$$F^T = a(t) \cdot F_o^M \cdot \cos\alpha$$

But, α changes with the length of the muscle!

Getting actuator length





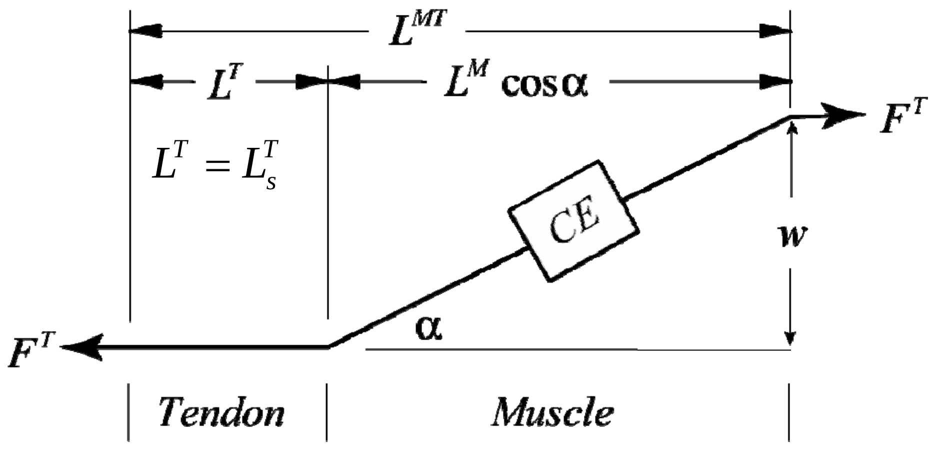
$$L^{MT} = \textit{path distance from muscle origin to insertion}$$

1b. Simplest with pennate fibers

1. $L^{MT} = L_s^T + \underline{L^M} \cos \underline{\alpha}$

2. $w = \underline{L^M} \sin \underline{\alpha} = L_o^M \sin \alpha_o$

Width is assumed to be constant.



Some algebra and trig…

$$\cos\alpha = \sqrt{\frac{\left(\frac{L^{MT} - L_s^T}{w}\right)^2}{1 + \left(\frac{L^{MT} - L_s^T}{w}\right)^2}}$$

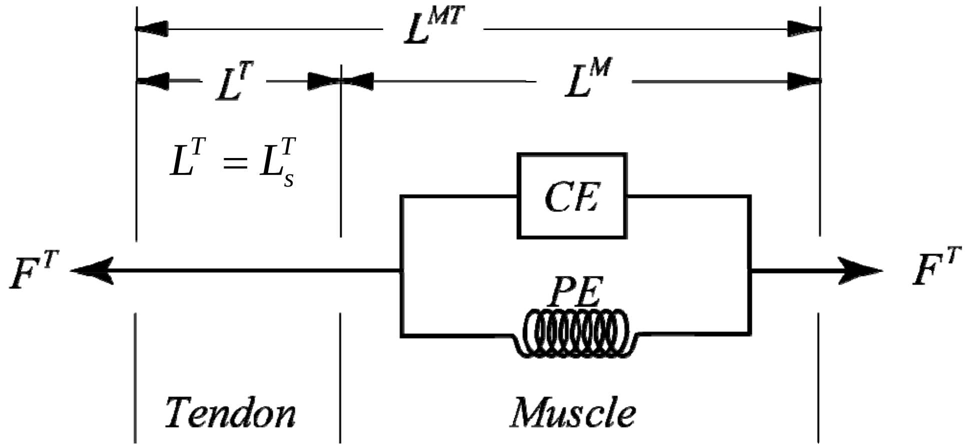
$$F^T = a(t) \cdot F_o^M \cdot \cos\alpha$$

2a. Force-length-velocity properties and inelastic tendon

- Assumptions
  - Tendon is inelastic
  - Dependence on length or velocity
  - Parallel fibered
- Parameters

  $F_o^M \quad L_o^M \quad L_s^T \quad \tilde{V}_{max}$
- Time-varying inputs

  $a \quad L^{MT} \quad \dot{L}^{MT}$

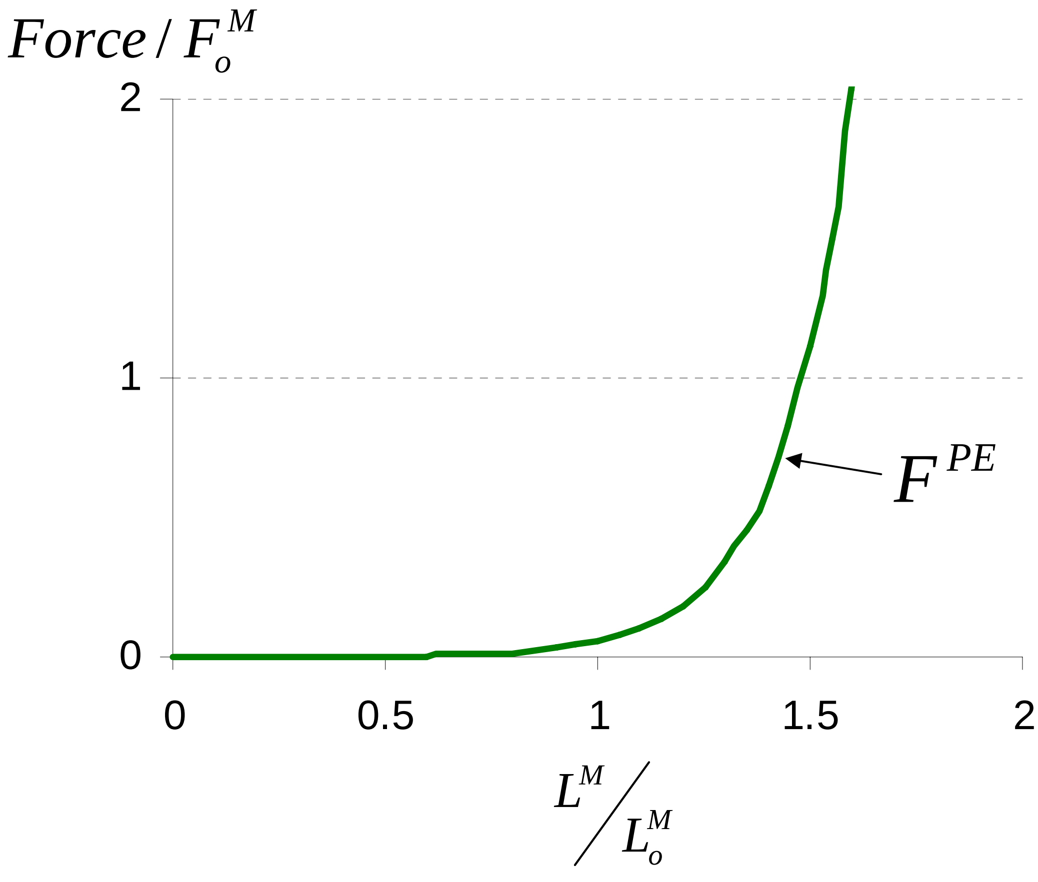


$$F^T = F^{CE} + F^{PE}$$

$$F^T = F^{CE}(L^M, \dot{L}^M) + F^{PE}(L^M)$$

2a. Force-length-velocity properties and inelastic tendon

$$F^{PE}(L^M) = F_o^M \cdot 3 \cdot 10^4 \cdot \exp[6 \cdot (\frac{L^M}{L_o^M} - 3.2)]$$

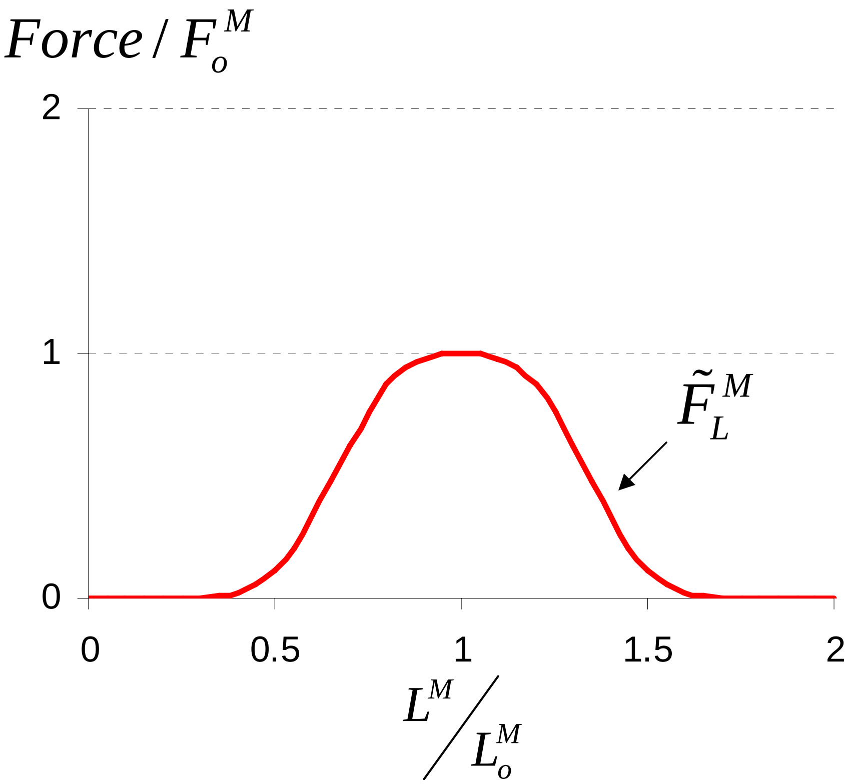


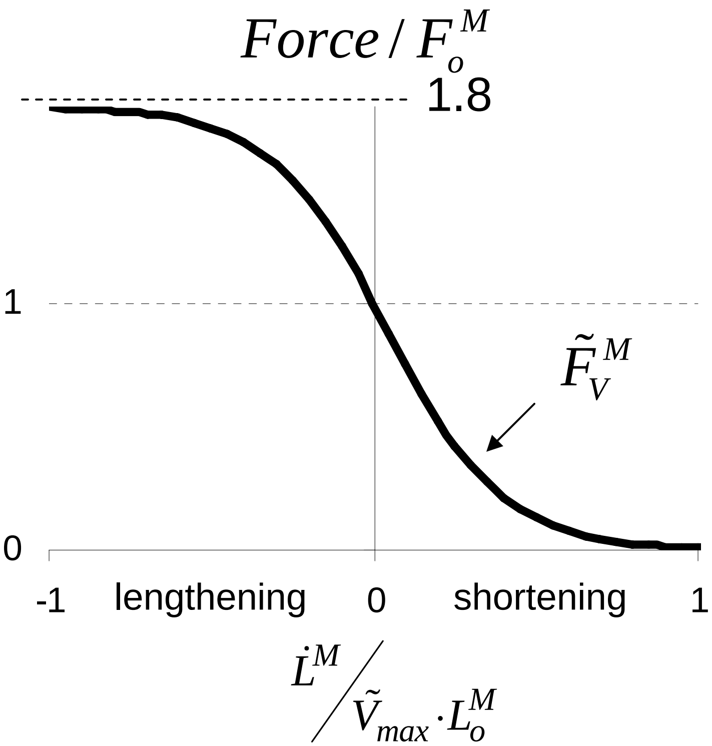
2a. Force-length-velocity properties and inelastic tendon

$$F^{CE} = a(t) \cdot F_o^M \cdot \underline{\tilde{F}_L^M(L^M)} \cdot \underline{\tilde{F}_V^M(\dot{L}^M)}$$

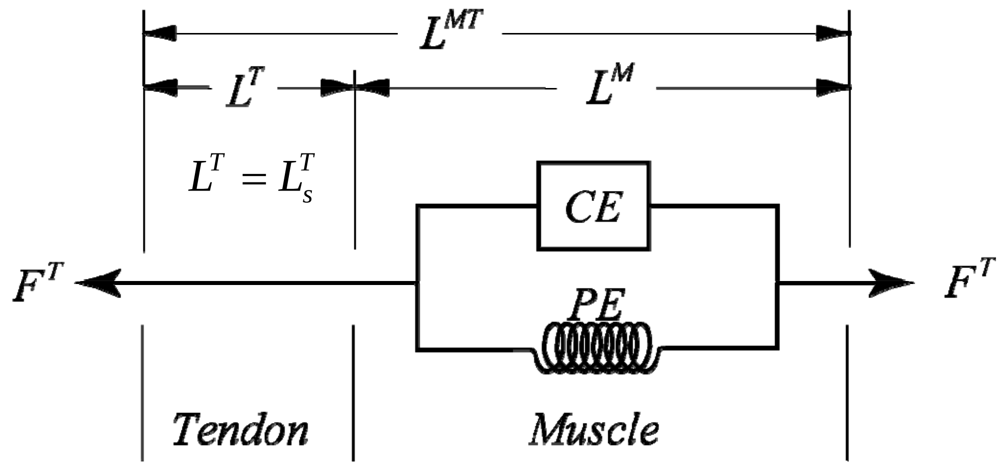
$$\tilde{F}_L^M(L^M) = \exp\left[\, 17.33 \cdot \left| \frac{L^M}{L_o^M} - 1.0 \right|^3 \right]$$

$$\tilde{F}_V^M(\dot{L}^M) = 1.8 - \frac{1.8}{1.0 + \exp\left[\frac{0.04 - \frac{\dot{L}^M}{\tilde{V}_{max} \cdot L_o^M}}{0.18} \right]}$$





2a. Force-length-velocity properties and inelastic tendon



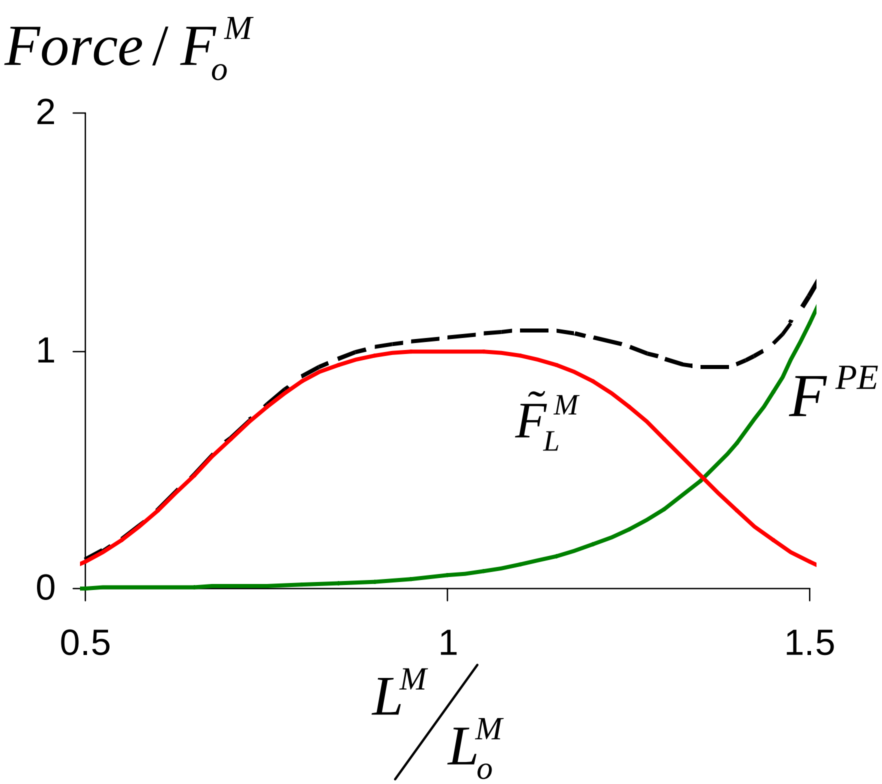
$$L^M = L^{MT} - L_s^T$$

$$\dot{L}^M = \dot{L}^{MT}$$

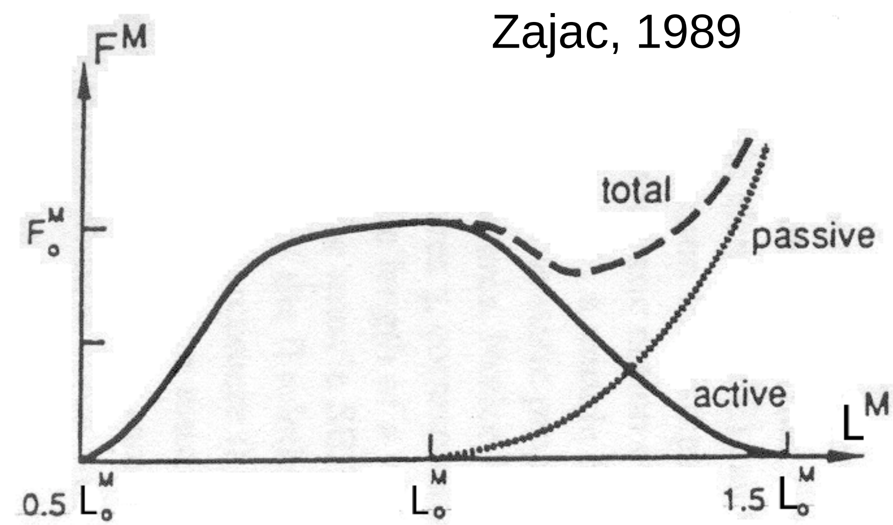
2a. Force-length-velocity properties and inelastic tendon

$$F^{PE}(L^M) = F_o^M \cdot 3 \cdot 10^4 \cdot \exp[6 \cdot (\tfrac{L^M}{L_o^M} - 3.2)]$$

$$\tilde{F}_L^M(L^M) = \exp[\, 17.33 \cdot \left|\tfrac{L^M}{L_o^M} - 1.0\right|^3 \,]$$

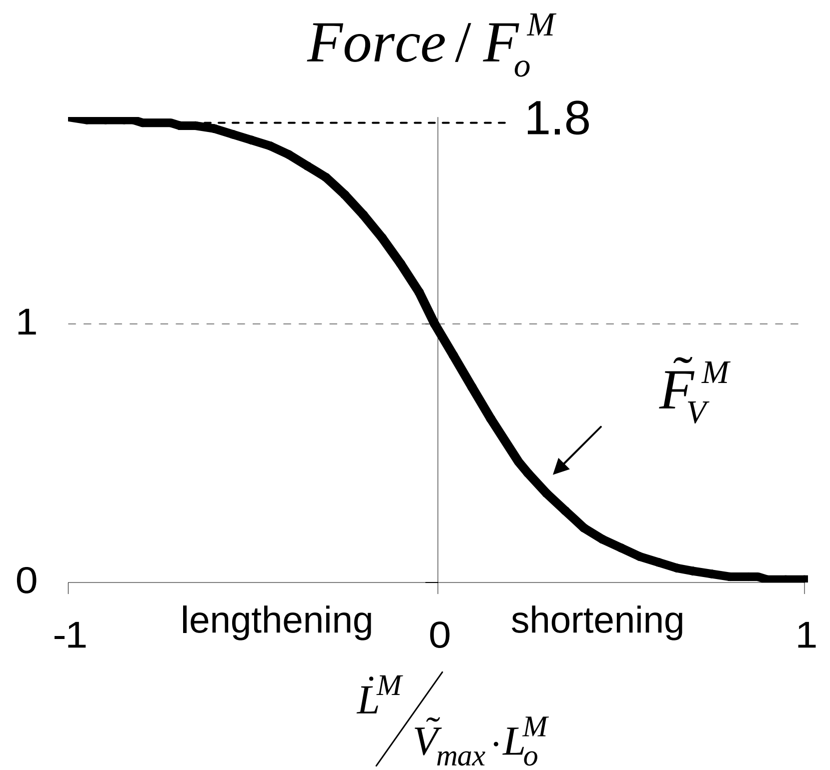


Zajac, 1989

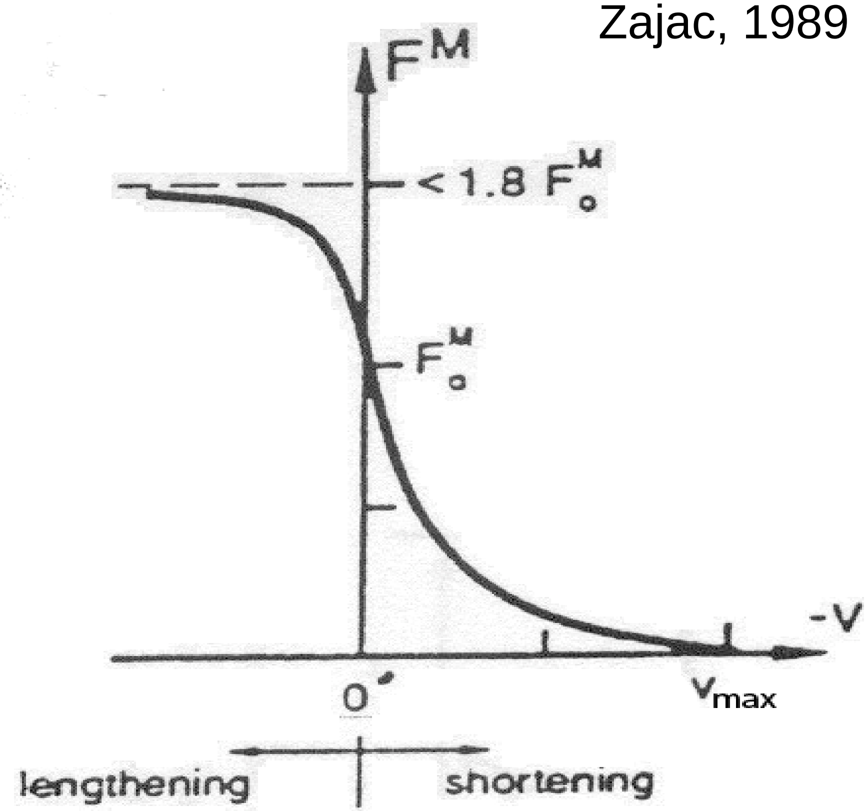


2a. Force-length-velocity properties and inelastic tendon

$$\tilde{F}_V^M(\dot{L}^M) = 1.8 - \frac{1.8}{1.0 + \exp\left[\frac{0.04 - \frac{\dot{L}^M}{\tilde{V}_{max} \cdot L_o^M}}{0.18}\right]}$$



Zajac, 1989

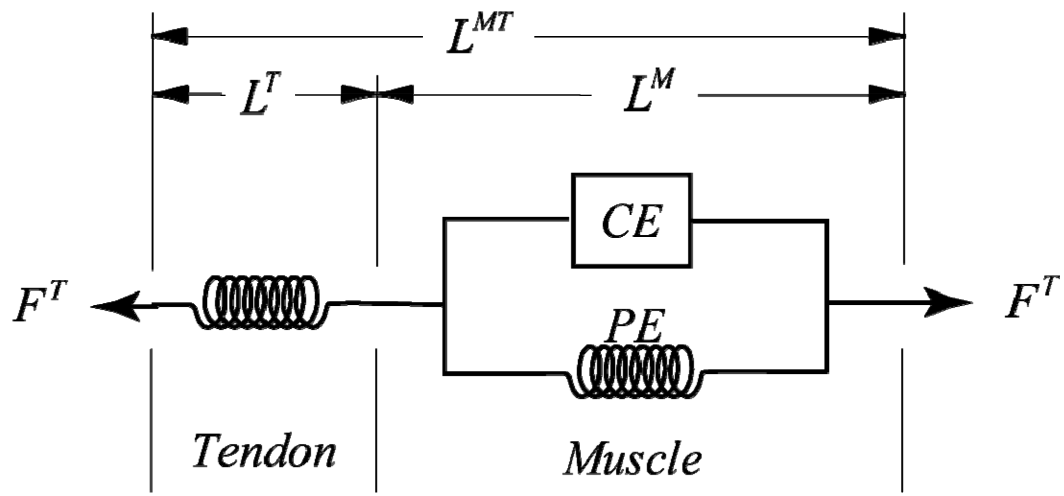


3a. Force-length-velocity properties and *elastic* tendon

- Assumptions
  - Tendon is elastic
  - Dependence on length or velocity
  - Parallel fibered
- Parameters

  $F_o^M \quad L_o^M \quad L_s^T \quad \tilde{V}_{max}$
- Time-varying inputs

  $a \quad L^{MT} \quad \dot{L}^{MT}$

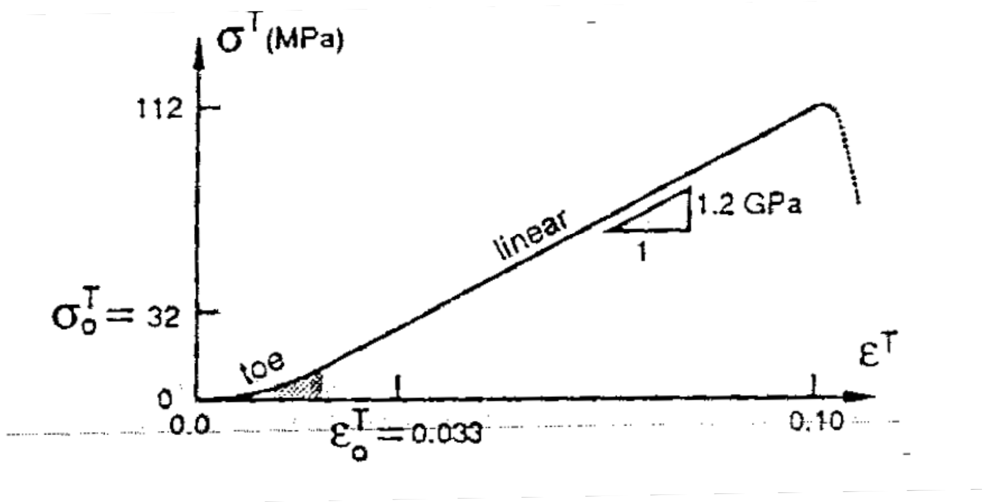


$$L^{MT} = L^T + L^M$$

$$\dot{L}^{MT} = \dot{L}^T + \dot{L}^M$$

A closed-form expression for F^T is generally not possible.

3a. Force-length-velocity properties and *elastic* tendon



$$F^T = F^T(t=0) + \int_{t=0}^{t} \dot{F}^T dt$$

$$F^T = k^T \cdot (L^T - L_s^T)$$

$$\dot{F}^T = k^T \dot{L}^T$$

$$\dot{F}^T = k^T (\dot{L}^{MT} - \dot{L}^M)$$

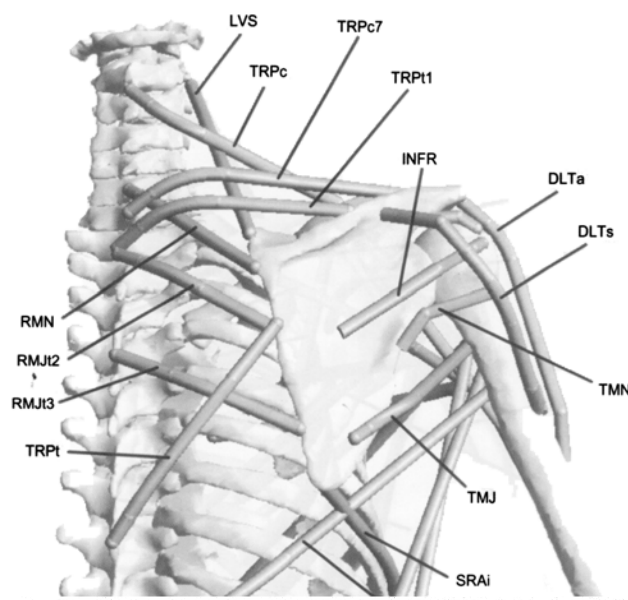
One last thing about tendon…

$$k^T \approx \frac{37.5 \cdot F_o^M}{L_s^T}$$

Where to get model parameters… the literature.

Garner & Pandy (2001)

100 B. A. GARNER AND M. G. PANDY



MUSCULOSKELETAL MODEL OF THE UPPER LIMB 105

TABLE I Architectural properties estimated for each musculotendon actuator in the upper-limb model: volume (Vol), maximum musculotendon length (L^{MT}_{max}), minimum musculotendon length (L^{MT}_{min}), physiological cross-sectional area (PCSA), optimal muscle-fiber length (L^{M}_{o}), tendon slack length (L^{T}_{s}), maximum isometric muscle force (F^{M}_{o}), and muscle pennation angle (α). Physiological cross-sectional area was defined as the ratio of muscle volume to optimal muscle-fiber length [35]

Muscle	Abbr.	Vol (cm^3)	L^{MT}_{max} (cm)	L^{MT}_{min} (cm)	PCSA (cm^2)	L^{M}_{o} (cm)	L^{T}_{s} (cm)	F^{M}_{o} (N)	α (deg)
subclavius	SBCL	8.80	7.15	6.28	4.36	2.02	5.07	144.02	0.00
serratus anterior (superior)	SRAs	92.20	12.24	5.84	8.12	11.35	0.27	268.05	0.00
serratus anterior (middle)	SRAm	71.71	19.32	11.23	4.00	17.91	0.75	132.12	0.00
serratus anterior (inferior)	SRAi	194.65	24.89	13.43	8.41	23.15	0.01	277.51	0.00
trapezius (C1–C6)	TRPc	116.23	20.12	9.46	6.24	18.62	0.48	205.95	0.00

Summary

- **Quantifying muscle and tendon force is important for understanding**
  - Performance
  - Bone and joint mechanics, development, and disease
  - Movement disorders.
- **The forces that muscles generate depend nonlinearly on length and shortening velocity.**
- **Lumped-parameter models vary in complexity**
  - The simplest can be developed using closed-form expressions
  - More complex models require ordinary differential equations
- **"Reasonable" models can be formulated based on a few parameters**

$$F_o^M \qquad L_o^M \qquad \alpha_o \qquad \tilde{V}_{max}^M \qquad L_s^T$$

Acknowledgements

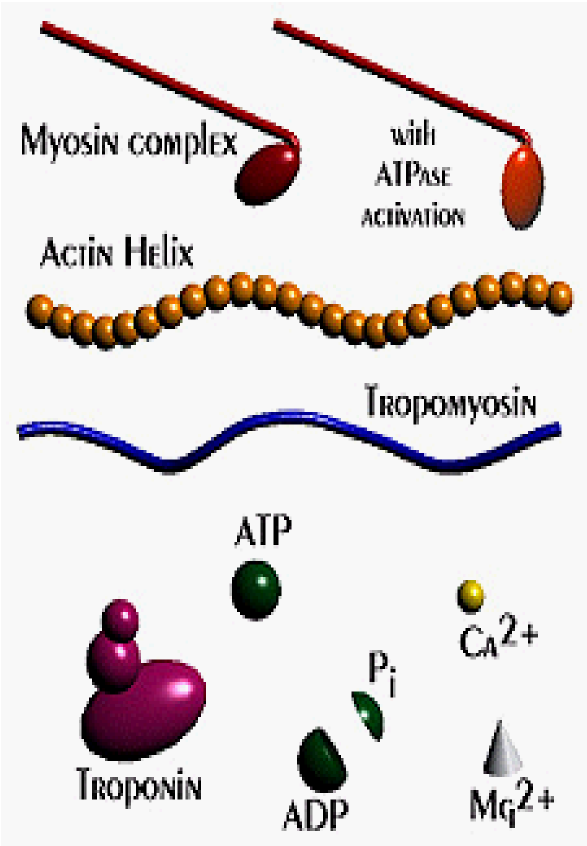
Supported by the National Institutes of Health through the NIH Roadmap for Medical Research Grant U54 GM072970.

NIH HD45109, HD38962, HD33929

Scaling

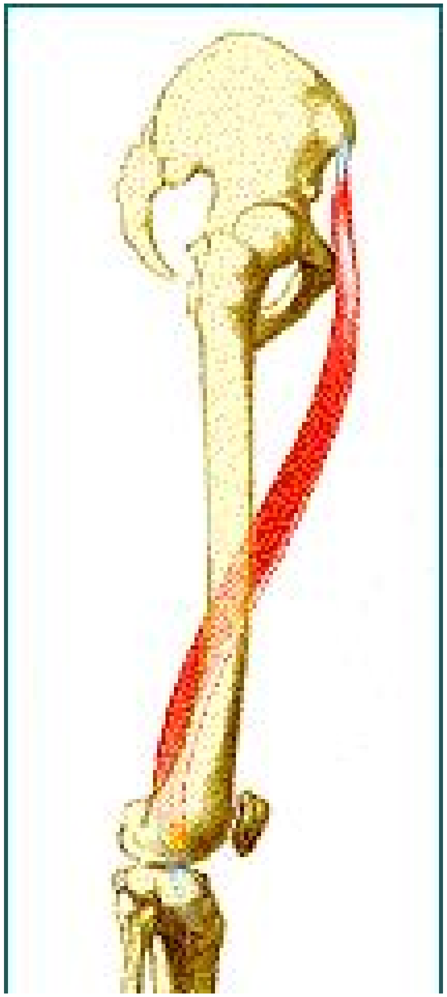
Muscle shortens as the proteins slide past each other

QuickTime™ and a
Cinepak decompressor
are needed to see this picture.



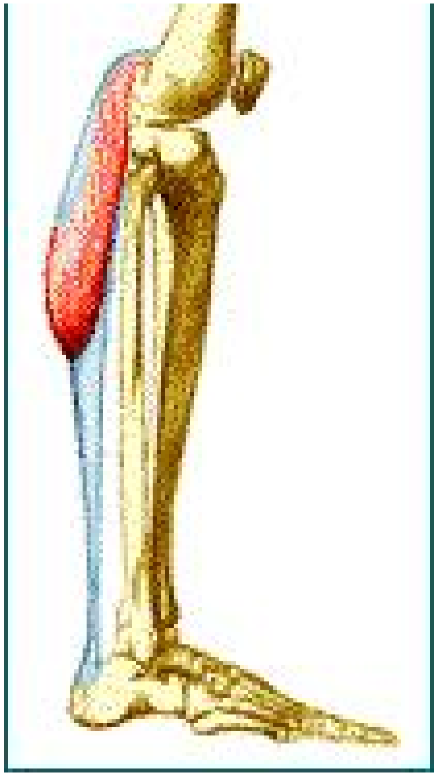
[http://www.sci.sdsu.edu/movies/actin_myosin_gif.html]

Muscle architecture

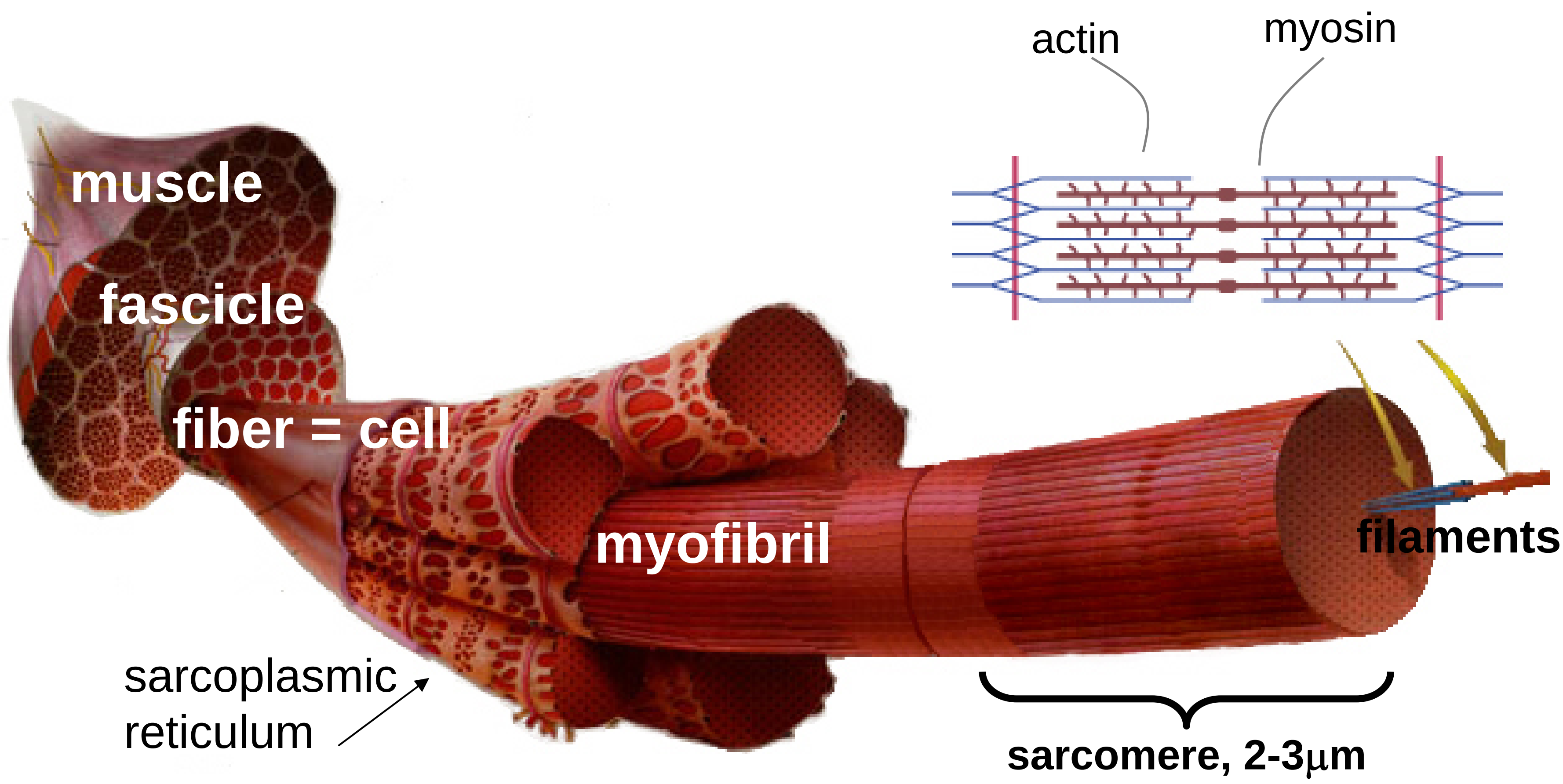


Sartorius has long, parallel fibers and very little tendon

Gastrocnemious has short, pennate fibers and a long tendon

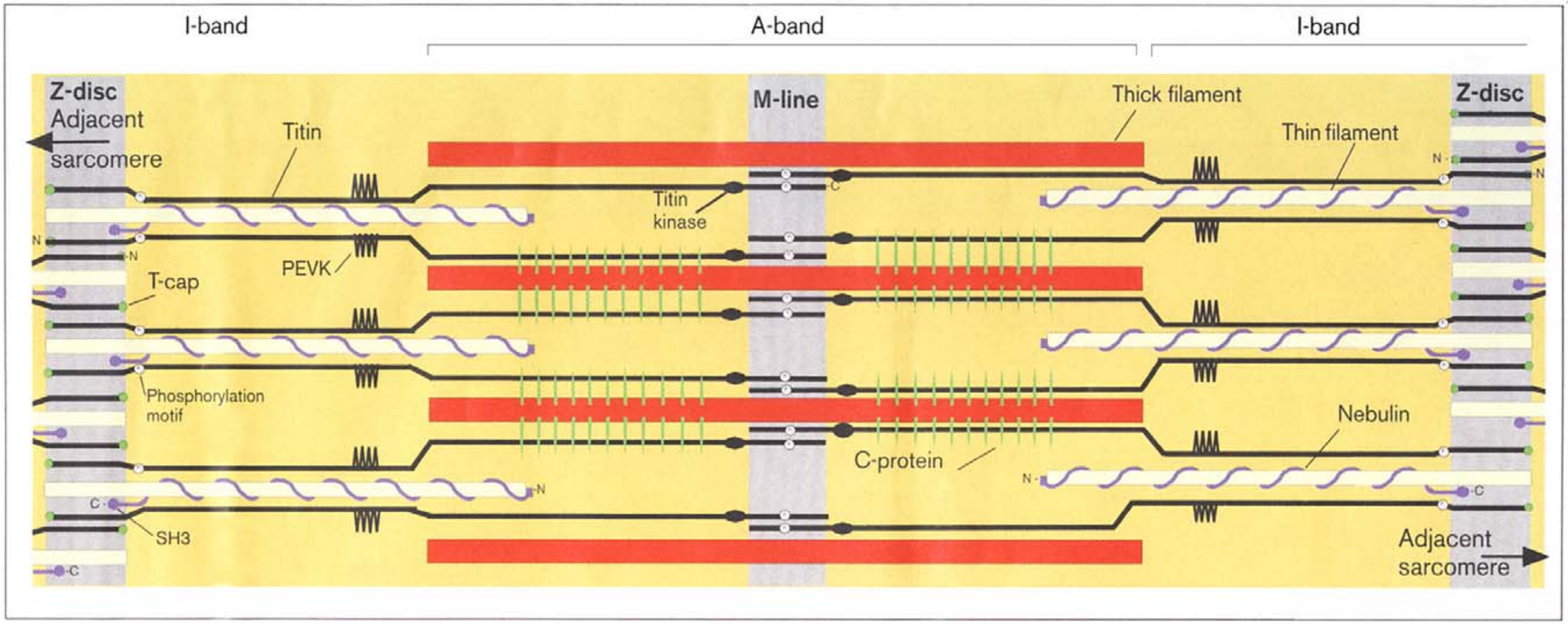


Hierarchical Muscle Structure



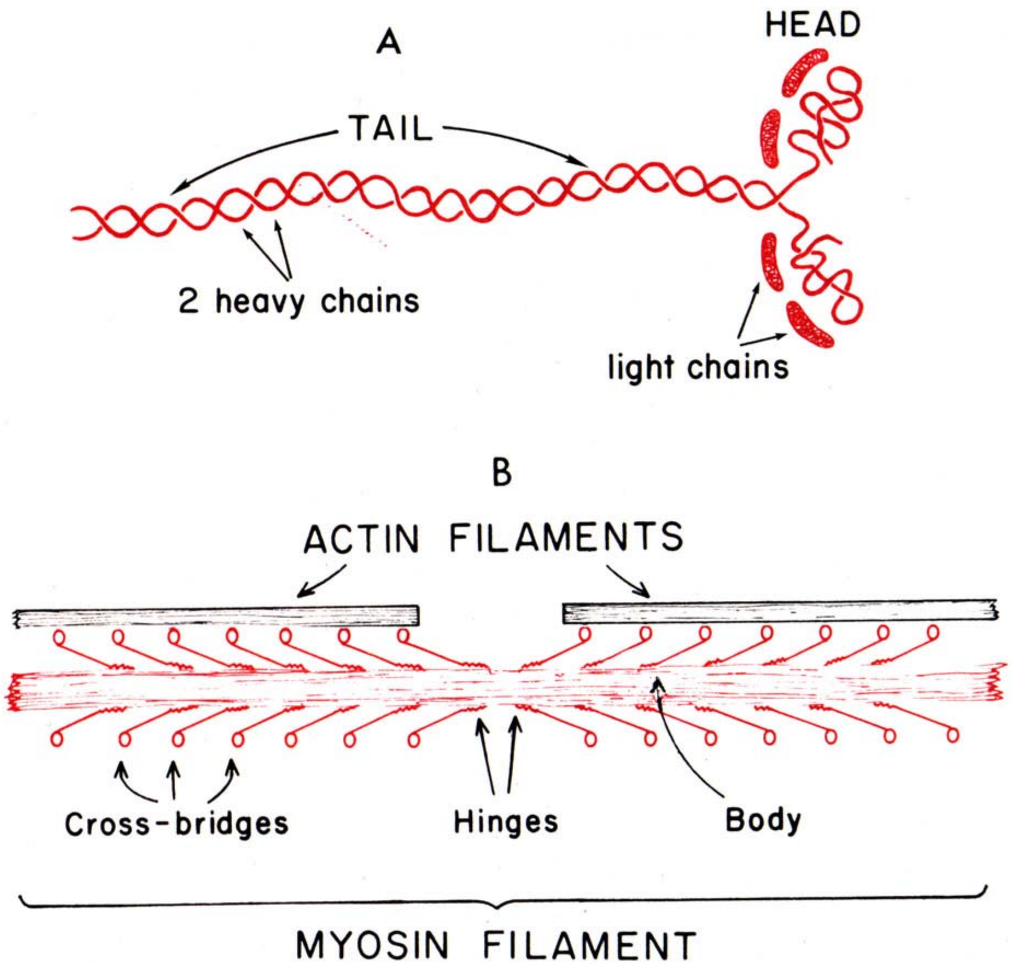
Adapted from Scientific American, September 2000

Sarcomere

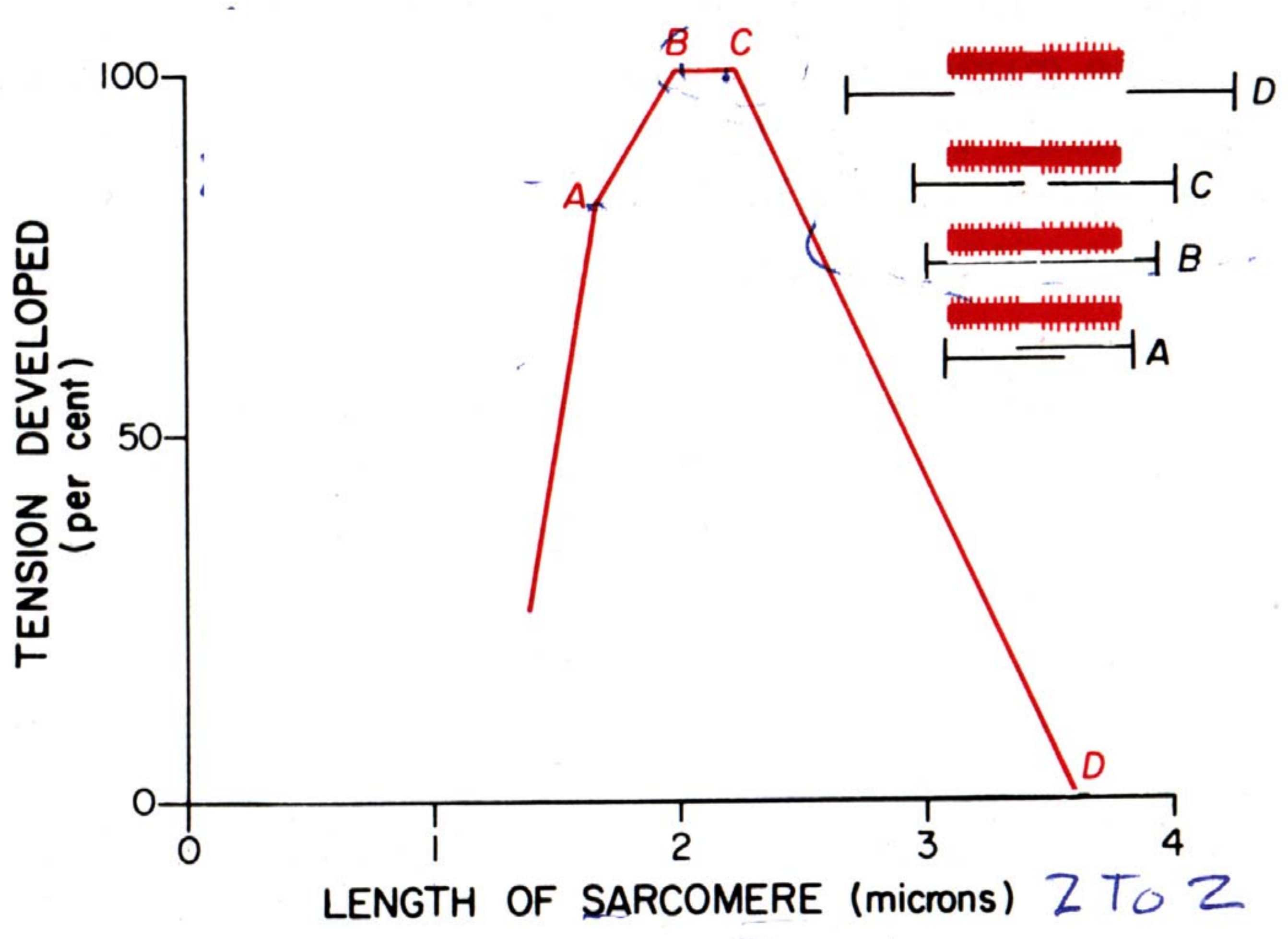


I-band
A-band
I-band
Z-disc
Adjacent sarcomere
Titin
M-line
Thick filament
Thin filament
Z-disc
Titin kinase
PEVK
T-cap
Phosphorylation motif
Nebulin
C-protein
SH3
Adjacent sarcomere

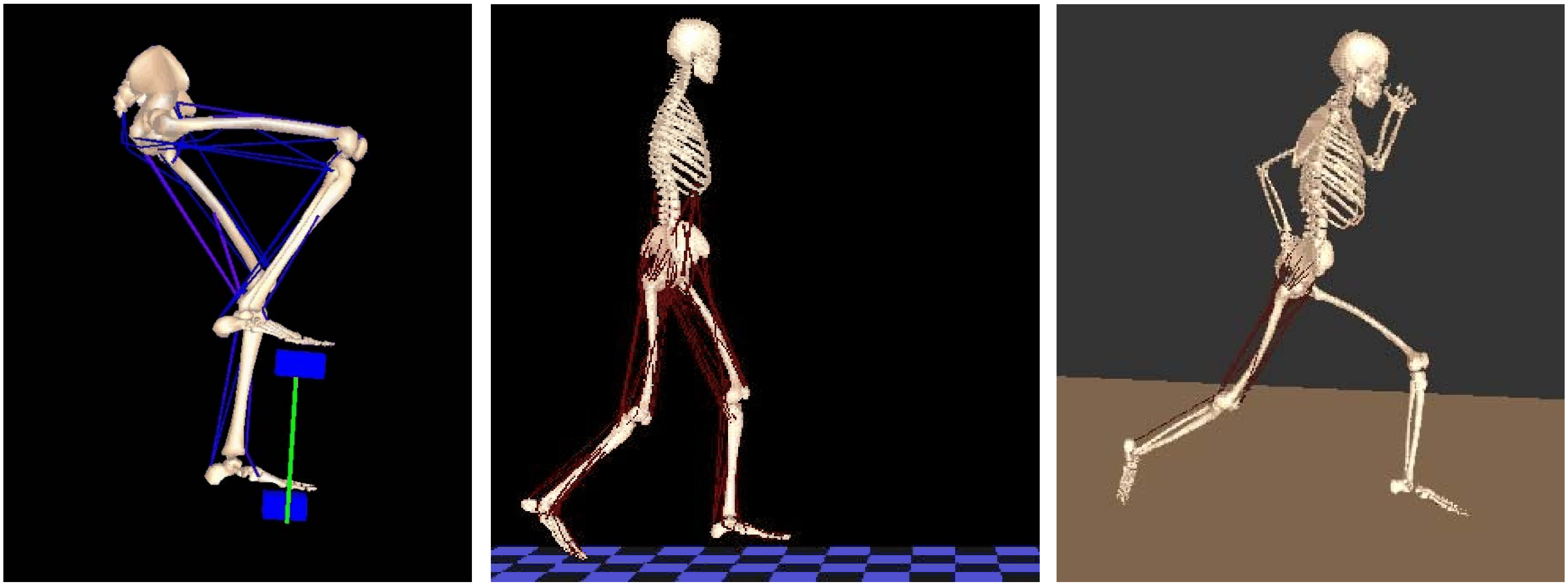
Actin-myosin cross-bridges



Tension developed by a sarcomere



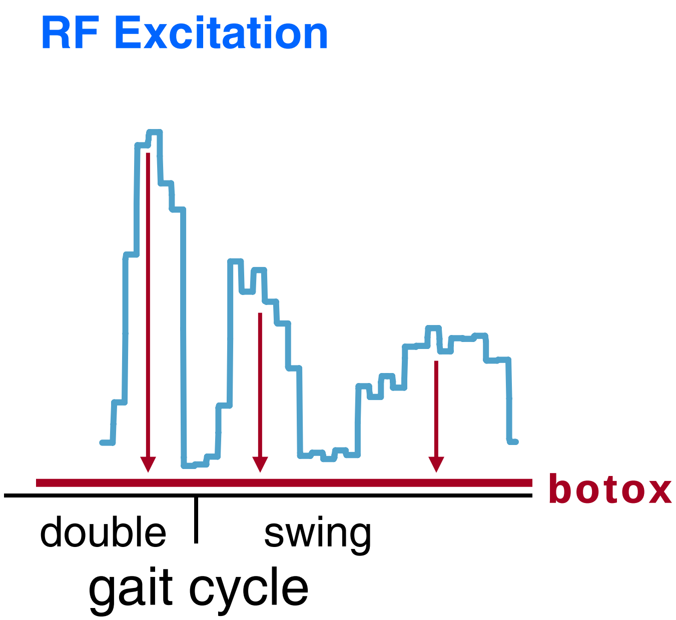
Simulations Generated with CMC



Each generated with less than 10 minutes of CPU time.

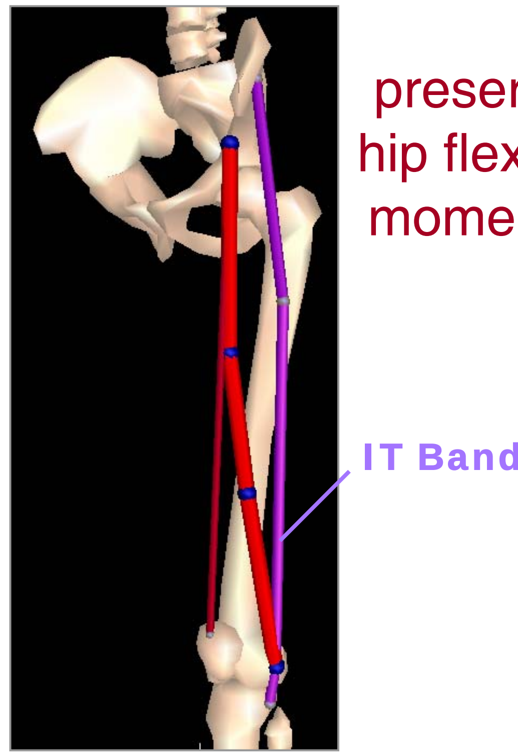
Simulated Treatments

Botulinum Toxin Injection



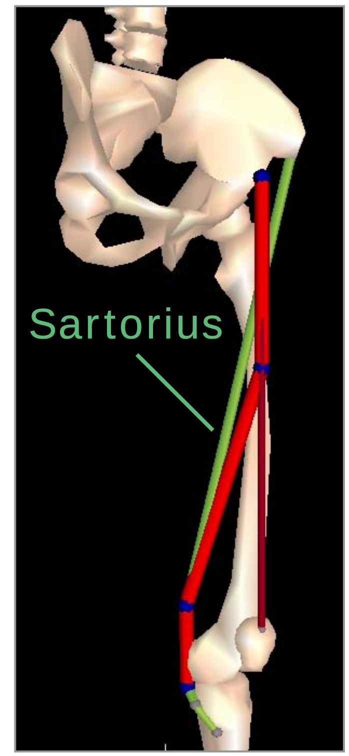
reduces both hip flexion knee extension moments

RF Transfers



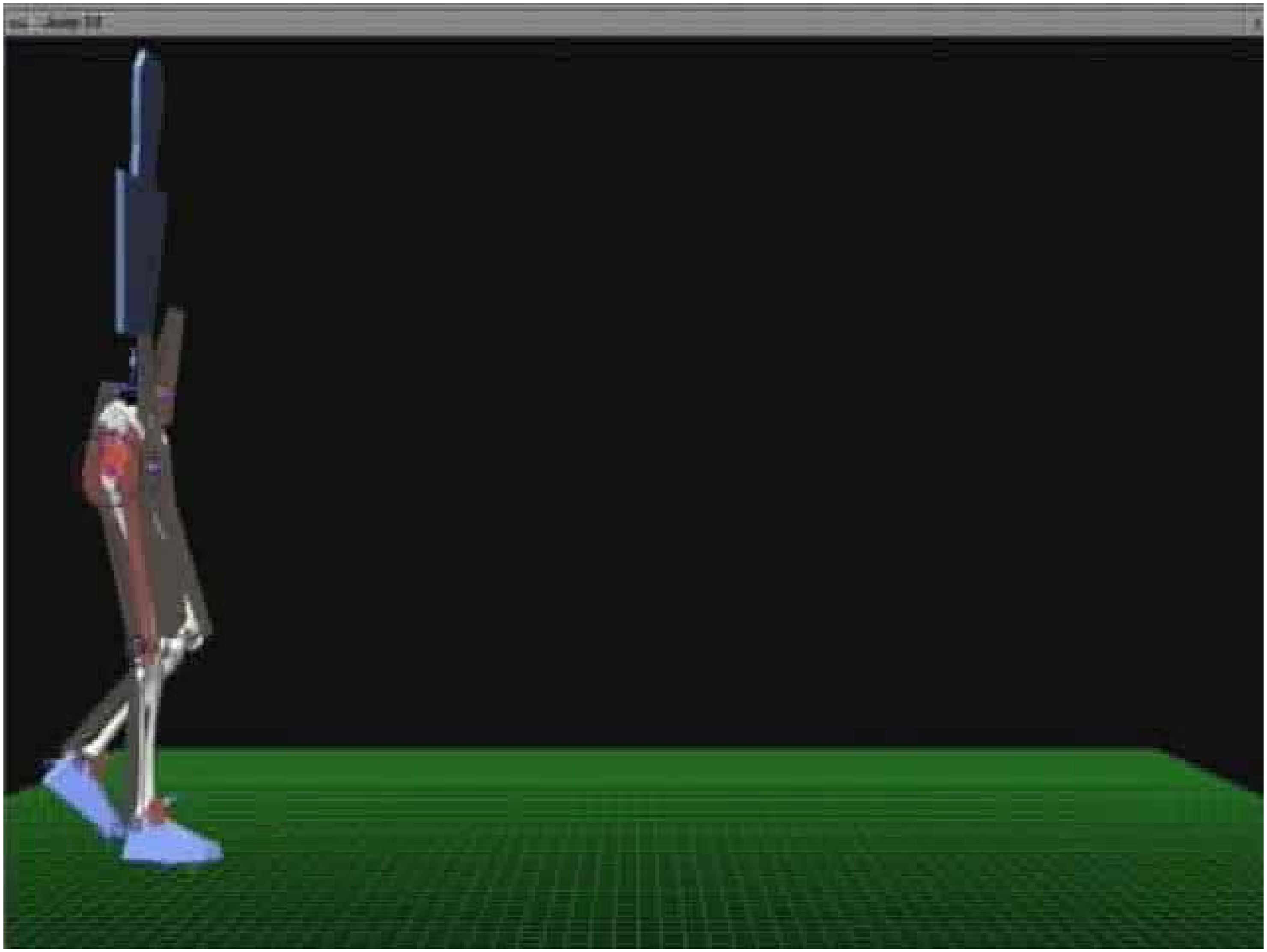
preserve hip flexion moments

eliminates knee extension moment



generates knee flexion moment

Simulations can help us understand muscle function



Simulations can help us understand muscle function