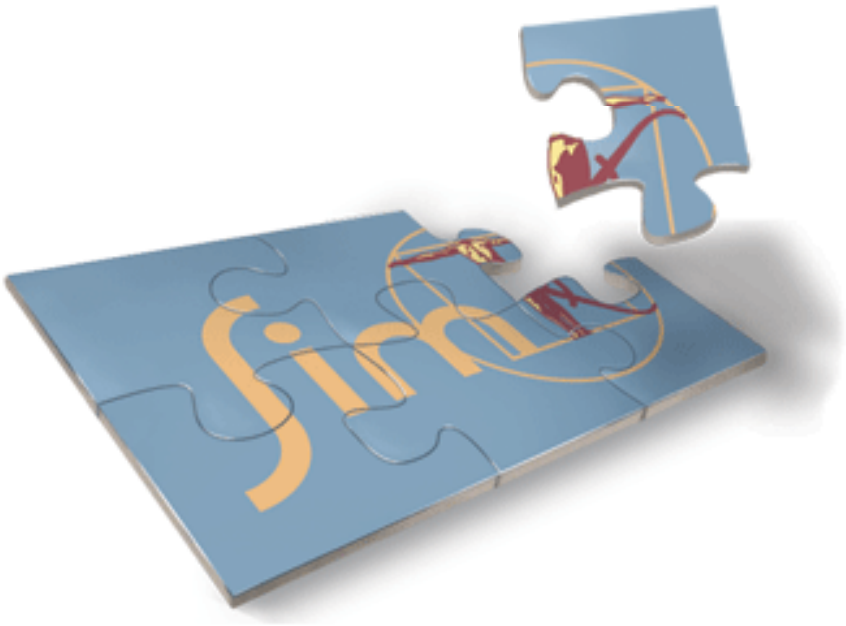
SimTK Documents



Simbody Advanced Programming Guide

Release 2.2

June, 2011

website: https://simtk.org/home/simbody

Copyright and Permission Notice

Acknowledgments

SimTK software and all related activities are funded by the Simbios National Center for Biomedical Computing through the National Institutes of Health Roadmap for Medical Research, Grant U54 GM072970. Information on the National Centers can be found at http://nihroadmap.nih.gov/bioinformatics.

Table of Contents

1 Introduction

Before reading this, you should already have read the Simbody and Molmodel User's Guide which you can find here: https://simtk.org/home/simbody, Documents tab. In the tutorial there, you learned how to build Simbody Systems out of Subsystems, MobilizedBodies, Constraints, and other objects. You can go a very long way doing nothing but that—building up Systems out of prewritten parts—but you may reach a point where it is not enough. Probably you will need to write specialized force elements. Perhaps you need a Constraint that does not correspond to any of the standard constraint types. Or maybe you need to model a connection that cannot be represented with the standard MobilizedBody classes. It's not enough to just use the classes provided by the toolkit; you need to write new ones. That is what this document will teach you how to do.

1.1 Extending Simbody

Let's begin by reviewing some things you learned in the tutorial. A System is made up of Subsystems. Each Subsystem can do any of the following:

1. Define state variables, which can be categorized into generalized coordinates (**q**), generalized speeds (**u**), auxiliary variables (**z**), and discrete variables (**d**).
2. Calculate information to be stored in the realization cache of a State object.
3. Calculate the time derivatives of continuous state variables.
4. Define constraint equations.
5. Define event trigger functions and event handlers.

You will learn how to write new Subsystems that may do any or all of these. This is the most general way that you can extend Simbody Systems with custom code. It often isn't the most convenient way, though. If all you want is to define one new constraint type, you shouldn't need to write an entire Subsystem. Simbody provides simpler mechanisms for extending a System in common ways. In fact, you have already seen one of them: rather than writing a new Subsystem to define an event handler, you simply write an EventHandler or EventReporter object, and then add it to the default Subsystem. You also can write custom subclasses of Force, Constraint, and MobilizedBody. We will see examples of all of these.

1.2 Realization Revisited

As you learned in the Simbody User's Guide, a State object holds two types of information: *state variables* and *cached results*. Cached results are stored in the *realization cache*, which is divided up into *stages*. Before you can access information in the cache, you must first make sure the State has been realized to the appropriate stage. For example, it must be at Position stage or later to access Cartesian coordinates, and at Velocity stage or later to access Cartesian velocities.

Stages can also be thought about in another way. Every state variable is associated with a particular cache stage:

Variable	Stage
t	Time
q	Position
u	Velocity
z	Dynamics
d	any

(A discrete state variable may be associated with any stage except Empty or Topology. When a Subsystem defines a discrete variable, it specifies what stage to associate it with.)

When a State is being realized to a particular stage, the values calculated and stored in the cache can only be those that depend on state variables for *that stage* or *earlier stages*. They may not depend in any way on state variables associated with later stages.

Why is this? Because whenever a state variable is modified, the cache is automatically reverted back to the stage immediately *before* the stage associated with that variable. If you modify a generalized coordinate q, the cache is reverted back to Time stage. If you modify a generalized speed u, the cache is reverted back to Position stage. This ensures that any information in the cache which might depend on that variable is discarded.

Suppose that a Subsystem failed to obey this rule. Suppose that, while realizing a State to Position stage, it made use of the generalized speeds. Then, at some later point, the speeds were modified. Those cached values would no longer be consistent with the state variables.

But because the Position stage cache entries would not be discarded, they would still be present in the cache and accessible to anyone who looked for them.

When you were simply using classes written by other people, you didn't have to worry about this. You simply trusted that information in the cache would always be correct. But now that you are preparing to write extensions to Simbody, you need to be aware of it. You need to know what promises are made by the System, and you have a responsibility to make sure your code does not break them.

1.3 The First Four Computation Stages

There are a total of ten computation stages. The later stages were discussed in the User's Guide, but very little mention was made of the first four stages: Empty, Topology, Model, and Instance. When running a simulation, there is usually not much reason to think about these stages, since all information you are likely to want is associated with one of the later stages. But when you are writing extensions to Simbody, these stages are very important. They are where the State gets constructed and configured to hold data. Let's examine each one of them.

Empty: This is the stage a newly constructed State object is in before it has been realized. It contains no information at all, and is not specific to any particular System.

Topology: When a State gets realized to Topology stage, it is configured to become a State for a particular System. In practice, this usually means allocating space in the cache for whatever data the System needs to store, and creating Model stage state variables. The Topology stage is unique in that no state variables may correspond to it. There is no such thing as a "Topology stage state variable" in a State. Logically, Topology stage "state variables" are the data members of the System; that is, they are stored with the System not with the State. The effect on the State when you realize it to Topology stage can only depend on properties of the System ("topological properties"), not on the value of any variable in the State.

Model: When a State is realized to Model stage, its complete set of state variables becomes fixed. This means that the set of state variables may depend on the value of a Model stage state variable. For example, Simbody allows rotations to be modeled with either quaternions or Euler angles. You select which representation to use by calling setUseEulerAngles() on the SimbodyMatterSubsystem. Your choice is stored in a Model stage discrete state variable. If

you select Euler angles, three generalized coordinates will be created for each rotation. If you select quaternions, there will be four. In general you use Model stage state variables (typically integers or boolean flags) to choose modeling options that may affect the number and type of later-stage state variables that are allocated.

This has an important consequence: if you change Model stage variables during a simulation (typically in an event handler), States created before the change may contain different state variables than ones created after. More commonly, Model stage variables are used to configure a System *before* beginning a simulation, not once the simulation has started.

Instance: At Instance stage, we know which force elements, constraints, and events are enabled, so the set of cache entries can be finalized. This means that the set of active forces, constraints, and event handlers may potentially change during a simulation. For example, Simbody uses an Instance stage variable to record which Constraints are disabled.

Instance stage state variables are often used for real-valued parameters, such as mass, geometry, spring constants, etc. Instance stage realization is a good time to precalculate values that won't change, or will only change during discrete events, not during continuous integration intervals.

2 Custom Forces, Constraints, and Mobilizers (joints)

Writing your own Subsystem as described in the next chapter provides the most flexibility, but there is almost always an easier way. Specifically, most built-in Simbody Subsystem manage a collection of "elements" that typically provide a great deal of functionality and customizability. The GeneralForceSubsystem, for example, has springs and the like but also a fully general "custom" force element that is easy to write. The SimbodyMatterSubsystem has built-in constraints and mobilizers (joints) but also provides the ability to write custom constraints and custom mobilizers.

2.1 A Custom Force

Writing a new Subsystem is the most general way to add custom features to a SimTK System, but it is not always the easiest way. Simbody provides special classes for writing common types of extensions: custom Forces, custom Constraints, and custom MobilizedBodies (meaning a custom joint). We will look at these classes in this chapter and see examples of how to use them.

In this section, we will implement a force that causes all the bodies in a MultibodySystem to repel each other. You will remember from the tutorial that Simbody's GeneralForceSubsystem provides a general mechanism for adding arbitrary Force elements to a System. Simbody provides a number of Force subclasses that implement common sorts of forces: Force::UniformGravity, Force::TwoPointLinearSpring, etc. It also provides a class called Force::Custom that can be used to define completely new forces; we'll use that here.

To use Force::Custom, you must write a subclass of Force::Custom::Implementation. Here is one that implements our global repulsion force:

```
class ExampleForce : public Force::Custom::Implementation {
public:
    ExampleForce(SimbodyMatterSubsystem& matter) : matter(matter) {
    }
```

```
    void calcForce(const State& state, Vector_<SpatialVec>& bodyForces,
            Vector_<Vec3>& particleForces, Vector& mobilityForces) const {
        for (MobilizedBodyIndex i(0); i < matter.getNumBodies(); i++) {
            const MobilizedBody& body1 = matter.getMobilizedBody(i);
            for (MobilizedBodyIndex j(0); j < i; j++) {
                const MobilizedBody& body2 = matter.getMobilizedBody(j);
                Vec3 r = body1.getBodyOriginLocation(state)-
                         body2.getBodyOriginLocation(state);
                Real distance = r.norm();
                Vec3 force = r/cube(distance);
                bodyForces[i][1] += force;
                bodyForces[j][1] -= force;
            }
        }
    }
    Real calcPotentialEnergy(const State& state) const {
        double energy = 0.0;
        for (MobilizedBodyIndex i(0); i < matter.getNumBodies(); i++) {
            const MobilizedBody& body1 = matter.getMobilizedBody(i);
            for (MobilizedBodyIndex j(0); j < i; j++) {
                const MobilizedBody& body2 = matter.getMobilizedBody(j);
                Vec3 r = body1.getBodyOriginLocation(state)-
                         body2.getBodyOriginLocation(state);
                energy -= 1.0/r.norm();
            }
        }
        return energy;
    }
    bool dependsOnlyOnPositions() const {
        return true;
    }
private:
    SimbodyMatterSubsystem& matter;
};
```

The calcForce() method is called to calculate the force. Notice that it has three different arguments for storing forces into: bodyForces, particleForces, and mobilityForces. Use bodyForces to apply Cartesian forces and torques to bodies. That is what we are doing in this example. You also can use mobilityForces to apply forces directly to individual degrees of freedom. That is, there is one scalar element corresponding to each generalized speed. A Force object may apply either or both types of force.

(Currently, particleForces is ignored. That is because Simbody does not yet support particles as a special case—you can include them as bodies, though. It is expected that they will be given special handling in a future version, so the interface includes them for forward compatibility.)

Similarly, calcPotentialEnergy() is called to calculate the potential energy due to the force. Finally, there is an optional method called dependsOnlyOnPosition(). The default implementation returns false. If you override it to return true, that enables an optimization to avoid recalculating the force and energy when a generalized speed or auxiliary state

variable is modified. Since our force depends only on **q**, not on **u** or **z**, we return true. This will potentially make our simulations run faster.

Here is how to add the custom force to a MultibodySystem:

```
MultibodySystem system;
SimbodyMatterSubsystem matter(system);
GeneralForceSubsystem forces(system);
Force::Custom(forces, new ExampleForce(matter));
```

As you see, it works just like any other Force object. We simply create a Force::Custom, passing an instance of our Implementation class as an argument. You can also write a "handle" class derived from Force::Custom which hides your force implementation class and provides a nicer API for your force element that acts exactly like built-in force elements do. See the Doxygen API documentation for Force::Custom for more information.

2.2 A Custom Constraint

Writing a custom Constraint is very similar to writing a custom Force: you create a class that extends Constraint::Custom::Implementation, then pass an instance to the constructor of a Constraint::Custom. But before showing an example, I need to give you warning: writing Constraints is significantly more difficult than writing Forces. It isn't that the programming interface is hard. There are several methods to implement, but that's not a big deal. The problem is that most constraints just involve a lot of math, and if your Constraint fails to work correctly, it can be difficult to figure out exactly where you made the mistake.

In principle, constraints are simple. As described in the User's Guide, a constraint is just an equation of the form $c(\mathbf{d};t,\mathbf{y}) = 0$. How hard can that be to implement? Actually, there are some constraints that really are as simple as that, and Simbody offers a special mechanism that lets you implement them in a truly easy way. We will discuss it in the next section. The problem is that, in many cases, the constraint function depends in some enormously complex way on a very large set of state variables.

Consider, for example, a simple Constraint::Rod (also known as a "distance constraint"). This specifies that the distance between points on two different bodies must remain fixed. The positions of those points depend on the positions and orientations of the two bodies. And those, in turn, depend on the generalized coordinates for the two bodies and for *every one of their parent bodies back to ground*. Trying to actually write the constraint equation explicitly would be completely impractical.

On its own, that isn't usually a problem. After all, Simbody will calculate the locations of the points for you, and it's easy enough to then calculate the distance between them. But there is a second issue that complicates matters. Each constraint equation also implies that its time derivatives are satisfied too. The Rod constraint, for example, generates three equations that must be satisfied: a position-level constraint equation requiring the distance between two points to be fixed; a velocity-level constraint equation requiring their relative velocity to be zero; and an acceleration-level constraint equation requiring their relative acceleration to be zero. You must implement all of these and make sure they are all consistent with each other. Again, Simbody can provide all the information you need, but deciding exactly how to put that information together correctly will take some math!

Constraints can be divided into three categories: *holonomic*, *nonholonomic*, and *acceleration-only*. A *holonomic* constraint is one defined at the position level (such as a Rod constraint). A holonomic constraint equation implies that its two time derivatives be satisfied as well, as discussed above. A *nonholonomic* constraint is defined at the velocity level. An example is Constraint::ConstantSpeed: it sets no restriction on the allowed values of coordinates, only on how those coordinates change with time. A nonholonomic constraint equation implies two conditions to satisfy (one at the velocity level and one at the acceleration level). An acceleration-only constraint equation is defined at the acceleration level, and implies only a single constraint condition to satisfy.

Let's take a look at an example. Here is a custom constraint that requires the distance between two bodies' origins to remain fixed. This is, of course, just a special case of the more general built-in Rod constraint. So this isn't a very useful class, but it is a fairly easy one to understand, so it makes a good example.

```
class ExampleConstraint : public Constraint::Custom::Implementation {
public:
    ExampleConstraint(MobilizedBody& b1, MobilizedBody& b2, Real distance) :
            Implementation(b1.updMatterSubsystem(), 1, 0, 0), distance(distance) {
        body1 = addConstrainedBody(b1);
        body2 = addConstrainedBody(b2);
    }
    Implementation* clone () const {
        return new ExampleConstraint(*this);
    }
    void realizePositionErrors(const State& state, int mp,
            Real* perr) const {
        Vec3 r1 = getBodyOriginLocation(state, body1, true);
        Vec3 r2 = getBodyOriginLocation(state, body2, true);
        perr[0] = ((r1-r2).normSqr()-distance*distance)/2;
    }
```

```
    void realizePositionDotErrors(const State& state, int mp,
            Real* pverr) const {
        Vec3 r1 = getBodyOriginLocation(state, body1, true);
        Vec3 r2 = getBodyOriginLocation(state, body2, true);
        Vec3 r = r2-r1;
        Vec3 v1 = getBodyVelocity(state, body1, true)[1];
        Vec3 v2 = getBodyVelocity(state, body2, true)[1];
        Vec3 v = v2-v1;
        pverr[0] = dot(v, r);
    }

    void realizePositionDotDotErrors(const State& state,
            int mp,  Real* paerr) const {
        Vec3 r1 = getBodyOriginLocation(state, body1, true);
        Vec3 r2 = getBodyOriginLocation(state, body2, true);
        Vec3 r = r2-r1;
        Vec3 v1 = getBodyVelocity(state, body1, true)[1];
        Vec3 v2 = getBodyVelocity(state, body2, true)[1];
        Vec3 v = v2-v1;
        Vec3 a1 = getBodyAcceleration(state, body1, true)[1];
        Vec3 a2 = getBodyAcceleration(state, body2, true)[1];
        Vec3 a = a2-a1;
        paerr[0] = dot(a, r) + dot(v, v);
    }

    void applyPositionConstraintForces(const State& state, int mp,
            const Real* multipliers, Vector_<SpatialVec>& bodyForcesInA,
            Vector& mobilityForces) const {
        Vec3 r1 = getBodyOriginLocation(state, body1, true);
        Vec3 r2 = getBodyOriginLocation(state, body2, true);
        Vec3 r = r2-r1;
        Vec3 force = multipliers[0]*r;
        addInStationForce(state, body2, Vec3(0), force, bodyForcesInA);
        addInStationForce(state, body1, Vec3(0), -force, bodyForcesInA);
    }
private:
    ConstrainedBodyIndex body1, body2;
    Real distance;
};
```

There's a lot to discuss here, so let's begin at the beginning. The constructor takes two bodies to constrain and the required distance between them. Notice the three integers that get passed to the superclass constructor. Those are the numbers of holonomic, nonholonomic, and acceleration-only constraint equations defined by this class. We are creating a single holonomic constraint equation, so we pass 1, 0, 0. Note that although there is a single constraint equation here, we are going to have to implement three routines—the equation itself and its first and second time derivatives.

The constructor calls addConstrainedBody() to register the MobilizedBodies the constraint acts on. This is an important optimization, since the cost of enforcing a constraint depends on the number of bodies involved (directly or indirectly). By telling Simbody what bodies are

constrained, you allow it to avoid calculations for degrees of freedom that have no effect on the constraint.

This optimization has a very profound impact on how you write constraints. If you look at the example above, you will notice that none of the calculation routines ever reference a MobilizedBody object, a MultibodySystem, or a SimbodyMatterSubsystem. Instead, Constraint::Custom::Implementation defines its own methods that you use instead, like getBodyOriginLocation() and getBodyVelocity(). These methods refer to bodies with a ConstrainedBodyIndex, not a MobilizedBodyIndex. If you don't see a method you need, don't figure out a clever loophole that lets you use SimbodyMatterSubsystem or MobilizedBody methods—that will not work correctly! Instead, post a question to the Simbody help forum at https://simtk.org/home/simbody, Advanced tab, Public Forums.

After you call addConstrainedBody() to register all of the constrained bodies, Simbody identifies an "ancestor body", which is the nearest common ancestor shared by all the constrained bodies. This allows it to define a "constrained system", consisting of the constrained bodies and all of their parents going back to the ancestor body. Often this will only be a small fraction of the bodies in the full System, but all the other bodies are guaranteed to have no impact on whether the constraint is satisfied. This can save a huge amount of computation. When you call getBodyOriginLocation(), it actually returns the location in the ancestor body's reference frame, *not* in the ground frame. But that doesn't really matter—in your code you just treat the ancestor as though it were ground.

Now look at the methods that implement the constraint. This is a holonomic constraint, so it involves three constraint equations. There is a virtual method corresponding to each one: realizePositionErrors(), realizePositionDotErrors(), and realizePositionDotDotErrors(). Each one calculates the error in the appropriate constraint equation. In this case, the position level equation is $(\mathbf{r}\cdot\mathbf{r}-d^2)/2=0$, the velocity level equation is $\mathbf{v}\cdot\mathbf{r}=0$, and the acceleration level equation is $\mathbf{a}\cdot\mathbf{r}+\mathbf{v}\cdot\mathbf{v}=0$. Each equation is just the time derivative of the previous one, and that is an absolute requirement! Note that you can't just produce some equivalent equation (like leaving out the 2 in the first equation here) because it is really the error term that we are returning and that is never zero. That is, the code returns only the left hand side of these equations and it is that left hand side that must be properly differentiated. That said, there are still many sets of equations that define the same constraint—for example, we could have used the actual distance $|\mathbf{r}|-d$ as the position level error, rather than

the difference of squares. That has some advantages, but simplicity of exposition is not among them!

In addition to calculating the constraint errors, we also need to calculate the constraint forces that should be applied at each time step to maintain the constraint. Simbody automatically calculates the Lagrange multipliers corresponding to each constraint and passes them to applyPositionConstraintForces(). We use them to calculate the force to apply to each body. (If you aren't familiar with Langrange multipliers, they are a little beyond the scope of this document, but you can easily find descriptions of them online. For our purposes, just think of them as the constraint forces that need to get generated in order to satisfy the constraint at the acceleration level.) We call addInStationForce(), which is a convenience method to apply a linear force at a specific point on a specific body. It works out the correct force and torque to apply, and adds them to the Vector of SpatialVecs.

Creating a nonholonomic or acceleration-only constraint is very similar. For a nonholonomic constraint, there are three virtual methods to implement: realizeVelocityErrors(), realizeVelocityDotErrors(), and applyVelocityConstraintForces(). For an acceleration-only constraint, there are two methods: realizeAccelerationErrors() and applyAccelerationConstraintForces(). Constraint::Custom::Implementation also has methods corresponding to each of the standard stages (realizeTopology(), realizeModel(), etc.), which can be overridden exactly as in a Subsystem. This allows you to define custom state variables and cache entries, which is useful if you want your constraint to depend on adjustable parameters.

2.3 Simple Constraints

Some constraints really are simple. Sometimes it is easy to write a function of the state variables that you want to constrain. In these cases, Simbody provides special classes that let you implement new constraints with very little work.

There are three special cases for which Simbody offers simple constraint classes. The first is a holonomic constraint that can be written as a simple function of the generalized coordinates: $c(\mathbf{q}) = 0$. This class is called Constraint::CoordinateCoupler, since it defines a coupling between some set of coordinates.

The second is a nonholonomic constraint that can be written as a simple function of the generalized coordinates and generalized speeds: $c(\mathbf{q}; \mathbf{u}) = 0$. This class is called

Constraint::SpeedCoupler. Although the constraint equation may involve coordinates, it is strictly a constraint on the speeds. It considers **q** to be fixed as suggested by the ";" above, and manipulates **u** to satisfy the equation given the current value of **q**.

The third case is a holonomic constraint that explicitly specifies the behavior of one generalized coordinate as a function of time: $q_i = f(t)$. This class is called Constraint::PrescribedMotion, since the motion of one coordinate is explicitly prescribed in advance.

Each of these classes requires you to provide a function of some set of state variables. This is done with the Function_<T> class. A Function object defines scalar or vector function of *m* arguments. That is, it provides a method with the following signature:

```
T calcValue(const Vector& x) const;
```

Function is a templatized class, with the output type as a template parameter. Most useful are types Real and short vector types like Vec3. All of the Constraint classes require a Function_<Real>, for which there is a typedef abbreviation Function. So calcValue() will return a Real.

Suppose we want a constraint that requires two generalized coordinates to always be equal to each other. This is done with a Constraint::CoordinateCoupler that enforces $c(\mathbf{q}) = q_1 - q_2 = 0$. Here is a Function class that implements $c(\mathbf{q})$:

```
class ConstraintFunction : public Function {
    Real calcValue(const Vector& x) const {
        return x[0]-x[1];
    }
    Real calcDerivative(const Array_<int>& derivComponents,
            const Vector& x) const {
        if (derivComponents.size() == 1)
            return derivComponents[0] == 0 ? 1 : -1;
        return 0;
    }
    int getArgumentSize() const {
        return 2;
    }
    int getMaxDerivativeOrder() const {
        return std::numeric_limits<int>::max();
    }
};
```

The implemention of calcValue() is simple: it just returns the difference between its two arguments. Since Function is templatized only on the type of the output value, not the number of input arguments, we also must implement getArgumentSize() to return the expected number of input arguments (2 in this case).

A Function class also must implement calcDerivative() to calculate the partial derivatives of the function. This takes a Array_<int> (behaves like std::vector<int>), which lists all arguments with respect to which to take the derivative. If the array is of length 1 (that is, a first derivative), we return either 1 or -1, depending on whether a derivative with respect to the first or second argument is requested. If the array length is greater than 1 (a second derivative or higher), we return 0.

A Function need not calculate all possible derivatives, since usually only the first few orders are required. It just needs to implement getMaxDerivativeOrder() to report the highest order derivative it can calculate. In this example there is no limit to which ones we can calculate (all derivatives higher than first order are 0), so we return the maximum possible integer value. The Functions used for constraints must support derivatives up to second order.

Here is how we add the Constraint to a System:

```
Array_<MobilizedBodyIndex> coordBody(2);
Array_<MobilizerQIndex> coordIndex(2);
coordBody[0] = body1.getMobilizedBodyIndex();
coordBody[1] = body2.getMobilizedBodyIndex();
coordIndex[0] = MobilizerQIndex(0);
coordIndex[1] = MobilizerQIndex(0);
Constraint::CoordinateCoupler constraint(matter, new ConstraintFunction(),
        coordBody, coordIndex);
```

In addition to telling the CoordinateCoupler what function to use, we also must tell it which coordinates to pass as arguments. For each coordinate, we specify the MobilizedBody it belongs to and the index of that coordinate for the MobilizedBody. In this example, we constrain the first coordinate of body1 to always equal the first coordinate of body2.

This example could actually be made even simpler. Simbody provides Function subclasses for common function types, such as linear functions, polynomials, and splines. Function::Linear represents a linear function of its arguments. For two arguments, for example, the function is *f*(x, y) = Ax+By+C. You provide the coefficients. We want (A, B, C) = (1, -1, 0), so we create the constraint as follows:

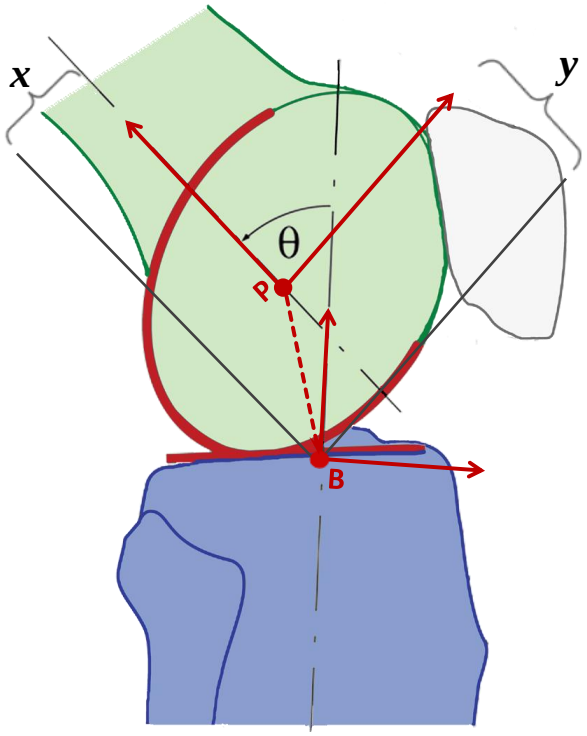
```
Vector coefficients(3);
coefficients[0] =  1;
coefficients[1] = -1;
coefficients[2] =  0;
Constraint::CoordinateCoupler constraint(matter,
        new Function::Linear(coefficients), coordBody, coordIndex);
```

Now we don't even need to write our own Function subclass! This is a Constraint that truly is easy to implement.

2.4 A Custom Mobilizer (The Easy Case)

Simbody also allows you to create new types of generalized coordinate joints, which are called "mobilizers" in Simbody terminology. Mobilizers are always instantiated at the time they are used to attach a body into the growing tree of bodies. Consequently there is no standalone mobilizer object; it always appears as a subclass of MobilizedBody which includes a Body and the mobilizer that attaches it to its parent body.

The ability to write a custom mobilizer is an extremely powerful feature and is unique to Simbody among multibody codes that we know of. It is particularly useful in biology where joints can be expected to undergo very complex motion not well-described by standard "pure" mechanical engineering joints like pin and ball joints. For example, the figure to the right shows a knee which has only a single degree of freedom but exhibits coupled rotational and translational motion. A custom mobilizer can model this with a single generalized coordinate and no constraints, with performance comparable to a simple Pin joint.



There is some good news and some bad news about this. First the bad news: in the general case, writing a custom MobilizedBody is even more difficult than writing a custom Constraint. Now the good news: as with Constraints, Simbody provides a class that lets you implement some, but not all, MobilizedBody types in a fairly easy way.

This time we'll start with the easy case. MobilizedBody::FunctionBased allows you to create new MobilizedBodies that have the following properties:

1. There is a fixed number of generalized coordinates **q** and the same number of generalized speeds **u** for this mobilizer.

2. There is a one-to-one correspondence between generalized coordinate derivatives and generalized speeds. That is, $dq_i/dt = u_i$ for each coordinate of the mobilizer (that is, $\dot{\mathbf{q}} = \mathbf{u}$ for this mobilizer).

3. The motion of the body can be described with six functions of the generalized coordinates, where three of them return translations along fixed axes, and the other three return rotation angles around fixed axes.

As an example, let's create a MobilizedBody with two generalized coordinates: a translation along X and a rotation around Y. I'm not sure exactly what a joint like that would be useful for, but I'll leave that up to you to decide! Let's see how to implement it.

First, we need six Functions. Two of them will be linear functions (for the X translation and Y rotation), and the other four will be constant functions that always return 0.

```
Vector coefficients(2);
coefficients[0] = 1;
coefficients[1] = 0;
Array_<const Function*> functions(6);
functions[0] = new Function::Constant(0, 0);
functions[1] = new Function::Linear(coefficients);
functions[2] = new Function::Constant(0, 0);
functions[3] = new Function::Linear(coefficients);
functions[4] = new Function::Constant(0, 0);
functions[5] = new Function::Constant(0, 0);
```

Notice that the linear functions are the second and fourth entries in the array. The first three functions return the rotation angles and the last three return the translations. By default, the axes are X, Y, and Z respectively, but you can modify them to have translations along arbitrary directions and rotations around arbitrary axes.

Next we need to tell Simbody what coordinates to pass to each function. We want to pass the first generalize coordinate to function 1, the second one to function 3, and no coordinates at all to the other functions. We create an Array_<int> (like std::vector<int>) for each function listing the coordinates to pass to it:

```
Array_<Array_<int> > coordIndices(6);
coordIndices[1].push_back(0);
coordIndices[3].push_back(1);
```

Now we can go ahead and create our custom MobilizedBody:

```
MobilizedBody::FunctionBased customBody(parent, body, 2, functions, coordIndices);
```

There are several other constructors that let you specify other options, such as the axes to use and the inboard and outboard transforms. Note that in general you can pass multiple coordinates into each function or you can pass the same coordinate into each function to create coupled rotational and translational motion driven by a common coordinate.

A common use of MobilizedBody::FunctionBased is to create joints based on experimental data. Consider the human knee shown above, for example. It has only one degree of freedom, but it rotates and translates in a complex way as a function of that degree of freedom. You can simply take experimental data describing the motion of a real knee, fit a set of splines to that data, and then use those splines to define a custom MobilizedBody.

If you are interested in a deeper understanding of custom mobilizers for biological and other complex joints, see this paper:

> A. Seth, M.A. Sherman, P. Eastman, S.L. Delp, "Minimal formulation of joint motion for biomechanisms," *Nonlinear Dynamics,* vol. 62, no. 1, pp. 291-303, 2010.

You can find this paper on the Simbody Documents page, or a link to the journal article on the Simbody Publications page.

2.5 A Custom Mobilizer (The Hard Case)

Many mobilizers can be implemented within the framework provided by MobilizedBody::FunctionBased, but not all. For example, if you want to represent a rotation with a quaternion, it will not work because that joint wouldn't satisfy the $\dot{\mathbf{q}} = \mathbf{u}$ condition we described above. Instead, you need to use MobilizedBody::Custom.

Here is a simple example of a custom MobilizedBody. Its behavior is identical to MobilizedBody::Translation: three generalized coordinates which are interpreted as displacements along the X, Y, and Z axes respectively. So it is not actually a useful class, but it makes a good example, since it is one of the simplest of all MobilizedBodies to implement.

```
class CustomTranslation : public MobilizedBody::Custom::Implementation {
public:
    CustomTranslation(SimbodyMatterSubsystem& matter) :
            Implementation(matter, 3, 3, 0) {
    }
    Implementation* clone() const {
        return new CustomTranslation(*this);
    }
    Transform calcMobilizerTransformFromQ(const State& s, int nq,
            const Real* q) const {
        return Transform(Vec3(q[0], q[1], q[2]));
    }
```

```
    SpatialVec multiplyByHMatrix(const State& s, int nu,
            const Real* u) const {
    return SpatialVec(Vec3(0), Vec3(u[0], u[1], u[2]));
    }
    void multiplyByHTranspose(const State& s, const SpatialVec& F, int nu,
            Real* f) const {
        Vec3::updAs(f) = F[1];
    }
    SpatialVec multiplyByHDotMatrix(const State& s, int nu,
            const Real* u) const {
        return SpatialVec(Vec3(0), Vec3(0));
    }
    void multiplyByHDotTranspose(const State& s, const SpatialVec& F, int nu,
            Real* f) const {
        Vec3::updAs(f) = Vec3(0);
    }
    void setQToFitTransform(const State&, const Transform& X_FM, int nq,
            Real* q) const {
        Vec3::updAs(q) = X_FM.p();
    }
    void setUToFitVelocity(const State&, const SpatialVec& V_FM, int nu,
            Real* u) const {
        Vec3::updAs(u) = V_FM[1];
    }
};
```

We pass three integers to the superclass constructor: the number of generalized speeds (3), the number of generalized coordinates (3), and the number of those coordinates that correspond to rotation angles (0).

The first method we need to implement is calcMobilizerTransformFromQ(). This is a straightforward method: given the list of generalized coordinates for this MobilizedBody, it calculates the mobilizer transform. In this example we simply take the three coordinates as the X, Y, and Z components of a translation:

```
return Transform(Vec3(q[0], q[1], q[2]));
```

This vector gives the origin of the mobilizer's M frame (on the MobilizedBody) as a vector from the origin of its F frame (on the parent body), expressed in the F frame. See the Simbody Tutorial for definitions of these frames, which are common to all mobilizers.

Next comes a group of four related methods: multiplyByHMatrix(), multiplyByHTranspose(), multiplyByHDotMatrix(), and multiplyByHDotTranspose(). The $\mathbf{H}$ matrix maps the generalized speeds $\mathbf{u}$ into the spatial velocity $\mathbf{V}$ introduced by the mobilizer: $\mathbf{V} = \mathbf{H}\cdot\mathbf{u}$. (A spatial velocity is a 2-vector of 3-vectors: $\mathbf{V}[0]$ is the angular velocity vector; $\mathbf{V}[1]$ is the linear velocity vector.) We need to provide methods to multiply by $\mathbf{H}$ (to convert $\mathbf{u}$ to $\mathbf{V}$) and $\mathbf{H}^T$ (which is also necessary). We also need to provide methods that multiply by the time derivative of this matrix, i.e., $\dot{\mathbf{H}}$ and $\dot{\mathbf{H}}^T$. In this example $\mathbf{H}$

conveniently does not change with time, so the latter two routines just return zero for all components.

Finally, there are two methods that "best fit" **q** and **u** based on a Transform or spatial velocity. These implement the standard MobilizedBody methods of the same names.

This MobilizedBody has a simple one-to-one relationship between generalized coordinates and generalized speeds, such that d**q**/dt = **u**. If this were not true, we would have to implement three additional methods: multiplyByNMatrix(), multiplyByNTranspose(), and multiplyByNDotMatrix(). The N matrix transforms the generalized speeds **u** into the derivatives of the generalized coordinates: d**q**/dt = N·**u**. The default implementations of these methods assume N is an identity matrix, so you only need to implement them when that is not true.

Like Constraints, MobilizedBody::Custom::Implementation has virtual methods you can implement to define new state variables, customize the appearance of the body, and various other things. See the API reference documentation for details.

3 A Custom Subsystem

A Subsystem is Simbody's most general element type. A System will typically contain a small number of Subsystems. The System's job is to dole out work to the Subsystems in a predefined order, and to permit Subystems to access one another's state variables and cache entries in a controlled fashion. Subsystems are not nested; they are a flat partitioning of the System's work. Typically a concrete System object will insist that certain types of Subsystems be present. MultibodySystem, for example, requires a SimbodyMatterSubsystem and a set of ForceSubsystems.

It is unusual to need a new Subsystem—be sure to check first whether you can achieve the results you want with custom forces, custom constraints, or custom mobilizers as discussed in the previous chapter. You may want to discuss your problem on the Simbody forum to see how others have tackled similar issues. But if you do need to make your own Subsystem, read on.

3.1 A First Subsystem

In this chapter, we will write a custom Subsystem. We will build it up in pieces, starting from the simplest possible Subsystem: one that does nothing at all.

```
#include "Simbody.h"
#include "SimTKcommon/internal/SubsystemGuts.h"

using namespace SimTK;

class ExampleSubsystemImpl : public Subsystem::Guts {
public:
    Subsystem::Guts* cloneImpl() const {
        return new ExampleSubsystemImpl();
    }
};

class ExampleSubsystem : public Subsystem {
public:
    ExampleSubsystem(MultibodySystem& system) {
        adoptSubsystemGuts(new ExampleSubsystemImpl());
        system.adoptSubsystem(*this);
    }
};
```

To understand this code, you first need to know that a Subsystem is actually defined by two different classes. Subsystem defines the "public interface" to it—those properties that most people access most of the time. There also are many properties related to the

implementation, which most users of the Subsystem do not care about most of the time. To keep the interface clean, these properties are split off into a separate class called Subsystem::Guts. An object of this type is created automatically for each Subsystem and can be accessed by calling getSubsystemGuts() on it.

To define a new type of Subsystem, you must create a subclass of each of these classes. The Subsystem subclass defines the public API, while the Subsystem::Guts subclass defines the implementation.

Now let's look at the example above. We define a class called ExampleSubsystemImpl that will provide our implementation. Since our Subsystem doesn't currently do anything, there isn't much to implement. The only method it is required to implement is cloneImpl(). There are many others which it *can* implement, and we will see some of those later. But all the others are only required if you need to provide certain features in your Subsystem.

ExampleSubsystem is equally simple. It defines only a constructor:

```
ExampleSubsystem(MultibodySystem& system) {
    adoptSubsystemGuts(new ExampleSubsystemImpl());
    system.adoptSubsystem(*this);
}
```

The Subsystem we are planning to create will work only with MultibodySystems, so we require one as a constructor argument. The constructor creates an ExampleSubsystemImpl, registers it by calling adoptSubsystemGuts(), and adds itself to the System.

You can now create an ExampleSubsystem and add it to a System exactly as you would any other Subsystem:

```
MultibodySystem system;
SimbodyMatterSubsystem matter(system);
ExampleSubsystem example(system);
```

Notice that the ExampleSubsystem is created as just a local variable. It will disappear as soon as that variable goes out of scope. The ExampleSubsystemImpl is the true persistent object. For this reason, you should never add fields to a Subsystem subclass or try to store information in it. Instead, store all information in the Subsystem::Guts subclass. A Subsystem object is merely a glorified pointer to it.

One other point is worth mentioning before we go on. Notice that we had to include an extra header file:

```
#include "SimTKcommon/internal/SubsystemGuts.h"
```

Why was this necessary? In the past, it has always been enough to just include Simbody.h, which includes all the header files you need. Usually, that is true. It includes all the headers that most people need most of the time. But it does not include headers that are needed only when writing extensions to SimTK. Those must be included separately.

3.2 A ForceSubsystem

Now let's make our Subsystem actually do something. We're going to implement in a Subsystem the same "mutual repulsion" capability we created in the previous chapter using a custom force element; if this is all you need to do you should definitely use a custom force, not a whole Subsystem! However, the example will serve nicely to illustrate the mechanics of building a Subsystem.

So once again we're going to cause all the MobilizedBodies in the System to repel each other with a force proportional to $1/r^2$:

```
#include "Simbody.h"
#include "simbody/internal/ForceSubsystemGuts.h"

using namespace SimTK;

class ExampleSubsystemImpl : public ForceSubsystem::Guts {
public:
    ExampleSubsystemImpl() : ForceSubsystem::Guts("Example", "1.0") {
    }
    Subsystem::Guts* cloneImpl() const {
        return new ExampleSubsystemImpl();
    }
    int realizeSubsystemDynamicsImpl(const State& state) const {
        const MultibodySystem& system = MultibodySystem::downcast(getSystem());
        const SimbodyMatterSubsystem& matter = system.getMatterSubsystem();
        Vector_<SpatialVec>& forces = system.updRigidBodyForces(state,
                Stage::Dynamics);
        for (MobilizedBodyIndex i(0); i < matter.getNumBodies(); i++) {
            const MobilizedBody& body1 = matter.getMobilizedBody(i);
            for (MobilizedBodyIndex j(0); j < i; j++) {
                const MobilizedBody& body2 = matter.getMobilizedBody(j);
                Vec3 r = body1.getBodyOriginLocation(state)-
                         body2.getBodyOriginLocation(state);
                Real distance = r.norm();
                Vec3 force = r/cube(distance);
                forces[i][1] += force;
                forces[j][1] -= force;
            }
        }
        return 0;
    }
```

```
    Real calcPotentialEnergy(const State& state) const {
        const MultibodySystem& system = MultibodySystem::downcast(getSystem());
        const SimbodyMatterSubsystem& matter = system.getMatterSubsystem();
        double energy = 0.0;
        for (MobilizedBodyIndex i(0); i < matter.getNumBodies(); i++) {
            const MobilizedBody& body1 = matter.getMobilizedBody(i);
            for (MobilizedBodyIndex j(0); j < i; j++) {
                const MobilizedBody& body2 = matter.getMobilizedBody(j);
                Vec3 r = body1.getBodyOriginLocation(state)-
                         body2.getBodyOriginLocation(state);
                energy -= 1.0/r.norm();
            }
        }
        return energy;
    }
};

class ExampleSubsystem : public ForceSubsystem {
public:
    ExampleSubsystem(MultibodySystem& system) {
        adoptSubsystemGuts(new ExampleSubsystemImpl());
        system.addForceSubsystem(*this);
    }
};
```

The first thing to notice is that we are using different parent classes: ForceSubsystem and ForceSubsystem::Guts. ForceSubsystem is a subclass of Subsystem defined by Simbody, which you should use for any Subsystem that applies forces to bodies. This ensures that Subsystems will be realized in the proper order, and also defines a calcPotentialEnergy() method in which you can calculate your force's contribution to the potential energy of the system. Also notice that we add the Subsystem to the System by calling addForceSubsystem(), instead of adoptSubsystem() like we did in the previous example.

We have added a method to ExampleSubsystemImpl, realizeSubsystemDynamicsImpl(). This will be called each time a State is being realized to Dynamics stage. There are similar methods for all the other stages. Each one has a default implementation that does nothing, so you only have to implement the ones you need.

Very little of the code in this method should look unfamiliar. We begin by calling getSystem() to get a reference to the System this Subsystem is part of, and use MultibodySystem's downcast() method which verifies that we have the expected type of System before returning a reference to it:

```
const MultibodySystem& system = MultibodySystem::downcast(getSystem());
```

We look up its SimbodyMatterSubsystem, then call updRigidBodyForces() to get a writeable reference to the vector of spatial forces acting on the bodies:

```
Vector_<SpatialVec>& forces = system.updRigidBodyForces(state, Stage::Dynamics);
```

Now we have a pair of nested loops over each pair of bodies. We calculate the displacement between them, use that to calculate a force, and add it to the appropriate entries in the vector.

There is a very similar method called calcPotentialEnergy(). This method is defined by ForceSubsystem::Guts. When someone calls calcPotentialEnergy() on a MultibodySystem, it loops over each of its ForceSubsystems, calls calcPotentialEnergy() on each one, and returns the sum of their potential energies.

Note that the above routines are very similar to the ones we wrote for a custom force in the previous chapter, but we have additional bookkeeping to do here.

3.3 Adding Parameters to the Subsystem

The force we implemented in the last section resembles Coulomb repulsion: lots of charged bodies all repelling each other. Wouldn't it be useful if you could change the strength of the force (to represent changing the charge on each body)? Of course, you could just edit the source code and recompile, but what if you want to run lots of simulations, each with a different strength for the force? It would be much easier if you could set the strength programmatically.

Here is a version that allows that:

```
class ExampleSubsystemImpl : public ForceSubsystem::Guts {
public:
    ExampleSubsystemImpl() : ForceSubsystem::Guts("Example", "1.0"),
            defaultForceStrength(1.0) {
    }
    Subsystem::Guts* cloneImpl() const {
        return new ExampleSubsystemImpl();
    }
```

```
    int realizeSubsystemDynamicsImpl(const State& state) const {
        const MultibodySystem& system = MultibodySystem::downcast(getSystem());
        const SimbodyMatterSubsystem& matter = system.getMatterSubsystem();
        Vector_<SpatialVec>& forces = system.updRigidBodyForces(state,
                Stage::Dynamics);
        for (MobilizedBodyIndex i(0); i < matter.getNumBodies(); i++) {
            const MobilizedBody& body1 = matter.getMobilizedBody(i);
            for (MobilizedBodyIndex j(0); j < i; j++) {
                const MobilizedBody& body2 = matter.getMobilizedBody(j);
                Vec3 r = body1.getBodyOriginLocation(state)-
                         body2.getBodyOriginLocation(state);
                Real distance = r.norm();
                Vec3 force = defaultForceStrength*r/cube(distance);
                forces[i][1] += force;
                forces[j][1] -= force;
            }
        }
        return 0;
    }
    Real calcPotentialEnergy(const State& state) const {
        const MultibodySystem& system = MultibodySystem::downcast(getSystem());
        const SimbodyMatterSubsystem& matter = system.getMatterSubsystem();
        double energy = 0.0;
        for (MobilizedBodyIndex i(0); i < matter.getNumBodies(); i++) {
            const MobilizedBody& body1 = matter.getMobilizedBody(i);
            for (MobilizedBodyIndex j(0); j < i; j++) {
                const MobilizedBody& body2 = matter.getMobilizedBody(j);
                Vec3 r = body1.getBodyOriginLocation(state)-
                         body2.getBodyOriginLocation(state);
                energy -= defaultForceStrength/r.norm();
            }
        }
        return energy;
    }
    void setDefaultForceStrength(Real strength) {
        defaultForceStrength = strength;
        invalidateSubsystemTopologyCache();
    }
    Real getDefaultForceStrength() const {
        return defaultForceStrength;
    }
private:
    Real defaultForceStrength;
};

class ExampleSubsystem : public ForceSubsystem {
public:
    ExampleSubsystem(MultibodySystem& system) {
        adoptSubsystemGuts(new ExampleSubsystemImpl());
        system.addForceSubsystem(*this);
    }
    void setDefaultForceStrength(Real strength) {
        updImpl().setDefaultForceStrength(strength);
    }
    Real getDefaultForceStrength() const {
        return getImpl().getDefaultForceStrength();
    }
```

```
private:
    ExampleSubsystemImpl& updImpl() {
        return dynamic_cast<ExampleSubsystemImpl&>(updRep());
    }
    const ExampleSubsystemImpl& getImpl() const {
        return dynamic_cast<const ExampleSubsystemImpl&>(getRep());
    }
};
```

First look at ExampleSubsystemImpl. We have added a field called defaultForceStrength. Why "default"? Because in the next section we will introduce a state variable for storing the force strength, and the value stored in the Subsystem will simply be the default value for newly created States. We have added a pair of accessor methods for getting and setting it, and modified the force and energy calculation to use it.

There is one line that needs explanation. In setDefaultForceStrength(), we call invalidateSubsystemTopologyCache(). The default force strength is stored in the Subsystem. That means it is a topological property (logically a Topology stage "state variable"). Whenever you modify a topological property, it is very important that you call this method. It marks that topology has changed, and all existing States need to be realized from Topology stage onward. That way if you try to use an old State object without recalculating, you will get an error.

You might wonder why this is necessary. If a topological change actually affects the data stored in a State (such as the set of cache entries), obviously old States will no longer be valid, but why does it matter for a simple change to the force constant?

The answer is that it is *especially* important for changes like this, because that is the only way to catch a variety of errors. A topological change should only ever be made before the start of a simulation, not in the middle. Otherwise, a saved State from earlier in the simulation (created based on an old value of the force constant) could easily get passed to a routine that would try to analyze it based on the new value. This is a very insidious sort of bug, because there is no way to detect it by looking at the State object itself. Calling invalidateSubsystemTopologyCache() ensures that all such errors will be caught.

If this seems restrictive, don't worry. There's an easy solution, which we'll see in the next section: if you want to be able to change something in the middle of a simulation, make it a state variable instead of a topological property.

The changes to ExampleSubsystem are very simple. We added two accessor methods, which just invoke the corresponding methods of ExampleSubsystemImpl. For convenience, note that we created two methods for looking up the ExampleSubsystemImpl:

```
ExampleSubsystemImpl& updImpl() {
    return dynamic_cast<ExampleSubsystemImpl&>(updRep());
}
const ExampleSubsystemImpl& getImpl() const {
    return dynamic_cast<const ExampleSubsystemImpl&>(getRep());
}
```

Since we will be accessing it many times, this saves us from having to write a dynamic_cast every time.

3.4 Creating a State Variable

We really want to be able to change the force strength at any time, not just before starting a simulation. To do that, it needs to be stored in a state variable. The following example shows how to do it. (Some methods are omitted to avoid repeating code you have already seen.)

```
class ExampleSubsystemImpl : public ForceSubsystem::Guts {
public:

    ...

    int realizeSubsystemTopologyImpl(State& state) const {
        forceStrengthIndex = allocateDiscreteVariable(state, Stage::Dynamics,
                                 new Value<Real>(defaultForceStrength));
        return 0;
    }
    int realizeSubsystemDynamicsImpl(const State& state) const {
        const MultibodySystem& system = MultibodySystem::downcast(getSystem());
        const SimbodyMatterSubsystem& matter = system.getMatterSubsystem();
        Vector_<SpatialVec>& forces = system.updRigidBodyForces(state,
                Stage::Dynamics);
        Real forceStrength = getForceStrength(state);
        for (MobilizedBodyIndex i(0); i < matter.getNumBodies(); i++) {
            const MobilizedBody& body1 = matter.getMobilizedBody(i);
            for (MobilizedBodyIndex j(0); j < i; j++) {
                const MobilizedBody& body2 = matter.getMobilizedBody(j);
                Vec3 r = body1.getBodyOriginLocation(state)-
                         body2.getBodyOriginLocation(state);
                Real distance = r.norm();
                Vec3 force = forceStrength*r/cube(distance);
                forces[i][1] += force;
                forces[j][1] -= force;
            }
        }
        return 0;
    }
```

```
    void setForceStrength(State& state, Real strength) const {
        Value<Real>::updDowncast(updDiscreteVariable(state,
                forceStrengthIndex) = strength;
    }
    Real getForceStrength(const State& state) const {
        return Value<Real>::downcast(getDiscreteVariable(state,
                forceStrengthIndex));
    }
private:
    Real defaultForceStrength;
    mutable DiscreteVariableIndex forceStrengthIndex;
};
```

Let's start by looking at realizeSubsystemTopologyImpl(). The first thing to notice is that, unlike realizeSubsystemDynamicsImpl(), it receives a non-const reference to the State. Topology and Model are the only stages where the State can be modified during realization (such as by defining new state variables).

That may seem strange. Isn't the whole point of realization to modify the State and store new information in it? The answer is that all cache entries are mutable. They are exactly what the name suggests: a cache. The state variables are the true information. The cache entries can always be regenerated based on them. So a const State reference still allows you to modify the cache, but not the state variables. Logically nothing has changed when the cache is filled in—you could delete the whole thing and regenerate it any time from the information in the state variables.

Now look at what happens when we realize Topology stage:

```
forceStrengthIndex = allocateDiscreteVariable(state, Stage::Dynamics,
                            new Value<Real>(defaultForceStrength));
```

We call allocateDiscreteVariable() on the State to create a new discrete variable. There are three arguments: the state, the stage that should be invalidated when this variable's value is changed, and a Value object to hold the actual value, with a default value given. We specify Dynamics stage, since that is the earliest stage whose computations depend on this variable. Notice that Value is a template. A discrete state variable can hold any type of value, even a large data structure. It is not restricted to just real values the way continuous state variables are.

The return value from allocateDiscreteVariable() is an index, which we will use whenever we want to access the value. We store that index in a mutable field of ExampleSubsystemImpl.

I hope you just cried out in horror at that last sentence? If not, please reread it and think carefully about why it's such a shocking statement! A System (including all its Subsystems) is supposed to be immutable during a simulation. All mutable information is supposed to go into the State. And here we are, realizing a State object… and we *modified a field of the Subsystem?* Didn't we just violate that design principle? And what will happen when we realize a different State, and write a new value to the field?

In any other situation, you would be completely correct. Topology stage is the *only* time when we are allowed to modify the System during realization. And not just any change; only properties that are calculated as a result of realizing the State during realizeTopology(). Indices are calculated for the state variables and cache entries as they are allocated, and we need to store them for future reference.

But what happens when we realize another State? Won't the indices get overwritten with new values? That's where Topology stage is special. Remember, there are no Topology stage variables *in a State*. What happens in realizeTopology() can depend *only* on topological properties of the System, which means you will get exactly the same indices every time it is called (unless you modify a topological property, in which case your previously created States will become invalid anyway.) Mutable fields in a System (or in a Subsystem that is part of a System) are like Topology-stage cache variables—they are calculated but add no new information. Everything they depend on is present as Topology-stage "state variables", i.e. non-mutable data members of System and Subsystem objects. You can recalculate them any time and you'll always get the same values.

But wait a minute—what about Model stage? You can also allocate state variables then. Does that mean it's also alright to store indices in the System while realizing Model stage?

The answer is a very emphatic *NO!* What happens during Model stage can depend on state variables, so you might get different indices for different States. Instead, create a data structure to hold any indices that will be calculated at Model stage. Then at *Topology* stage, allocate a cache entry to hold an instance of that structure. That way, all the indices calculated at Model stage will be stored in the State, not the System.

Now look at how we access the state variable:

```
void setForceStrength(State& state, Real strength) const {
    Value<Real>::updDowncast(updDiscreteVariable(state,
            forceStrengthIndex)) = strength;
}
Real getForceStrength(const State& state) const {
    return Value<Real>::downcast(getDiscreteVariable(state,
            forceStrengthIndex));
}
```

We invoke updDiscreteVariable() to get a writable reference to an AbstractValue object, or getDiscreteVariable() to get a const reference. We then ask the concrete Value type to verify that we have the expected kind of value, then cast the result to the appropriate type.

Finally, realizeSubsystemDynamicsImpl() and calcPotentialEnergy() need to be modified to use the value stored in the state variable. This is trivial: we just call getForceStrength(), then use that value instead of the default value.

3.5 Other Subsystem Features

There are many other things a Subsystem can do. Some are trivial to implement, while others are rarely used, so rather than discuss extended examples of each one, we will simply go through them quickly in this section. You can get much more information by looking in the API reference documentation by class or method name.

3.5.1 Allocating continuous state variables q,u, and z

Allocating continuous state variables is similar to allocating discrete ones, but simpler. You invoke allocateQ(), allocateU(), or allocateZ() on the State to allocate a contiguous block of state variables. Each of these methods takes a Vector containing the initial values of the state variables to allocate. The length of the Vector determines how many variables will be allocated. The return value is the index of the start of the block within the Vector of state variables *for that Subsystem*. For example, if you allocate a block of **q**'s by writing

```
qindex = allocateQ(state, initialValues);
```

then you would look up the value of the first one by calling

```
value = getQ(state)[qindex];
```

If you allocate continuous state variables, you will usually also want to implement realizeSubsystemAccelerationImpl() to calculate their derivatives. You set them by calling updQDot(), updUDot(), and updZDot() on the State. For *q*'s you also need to provide second

time derivatives via updQDotDot(). Those are often, but not always, the same as udots. The qdot, udot, zdot, and qdotdot values are actually cache entries created and managed automatically by the State as a result of the allocation of the corresponding continuous variables.

3.5.2 Allocating your own cache entries

If a Subsystem needs to create a "discrete" cache entry, that works almost exactly like creating a discrete state variable. You call allocateCacheEntry() to create it, and getCacheEntry() or updCacheEntry() to access a value. However, a cache entry is associated with *two* stages: the earliest stage at which it *can* be realized (evaluated), and the latest stage at which it *will* be realized. The "earliest" stage is the highest stage of any state variable on which the cache entry's value depends. If any state variable at that stage or lower is modified, the cache entry is invalidated automatically. The "latest" stage is a promise that if a State has been realized to that stage or later then you can depend on the cache entry's value being valid. For example, a cache entry holding the value of an expensive calculation that depends on Position variables (e.g. *q*'s) but isn't normally needed until Dynamics stage can be declared with earliest=Position and latest=Dynamics. It should then be evaluated during realize(Dynamics), after which it will be presumed valid, and it will be automatically invalidated if any Position-affecting state variable (or earlier stage variable) is modified.

3.5.3 Allocating cache entries with "lazy" evaluation

It is useful to set the "latest" stage to Stage::Infinity, meaning that no promise is being made that the cache value will ever be realized. You can do that with allocateCacheEntry() but it is more clear if you use allocateLazyCacheEntry(). In either case the cache entry is "lazy" in the sense that it will only be realized if someone asks for its value. The first time after the "earliest" stage has been realized that someone asks for the value must cause realization to happen. After that the value is available for free. The method isCacheValueRealized() can be used to check whether the value has been calculated, and the method markCacheValueRealized() is used to mark the cache value valid any time between stage "earliest" and stage "latest". These methods should be used as follows, for a cache entry CE whose value type is CEType and whose CacheEntryIndex is CEIndex.

1. Allocate your lazy cache entry something like this:

```
CEIndex = allocateLazyCacheEntry(state,stage,new Value<CEType>());
```

2. Write a realizeCE() method structured like this:

```
void realizeCE(const State& s) const {
    if (isCacheValueRealized(s,CEIndex))
        return;
    // calculate the cache entry, update with updCacheEntry()
    markCacheValueRealized(s,CEIndex);
}
```

3. Write a getCE() method structured like this:

```
const CEType& getCE(const State& s) const {
    realizeCE(s); // make sure CE has been calculated
    return Value<CEType>::downcast(getCacheEntry(s,CEIndex));
}
```

4. Write an updCE() method like this:

```
CEType& updCE(const State& s) const {
    return Value<CEType>::updDowncast(updCacheEntry(s,CEIndex));
}
```

Note that all the above routines are const, even though they may modify the cache entry's value; that's because cache entries are always mutable since, as discussed above, they do not contain any new information.

3.5.4 Creating event handlers

Another feature of Subsystems is the ability to define event handlers. This is not a widely used feature, because it is usually easier to use the EventHandler and EventReporter classes. But sometimes you might prefer to have a Subsystem do its own event handling, especially if the events are closely related to other functions of the Subsystem.

To define a scheduled event or scheduled report, the Subsystem should call createScheduledEvent() when realizing Instance stage. This allocates a globally unique event ID that is thereafter used to refer to that event. Next, you need to implement one or both of the two methods calcTimeOfNextScheduledEvent() and calcTimeOfNextScheduledReport(). These methods return the next time at which any events or reports will occur, and the IDs of all events/reports that will occur at that time. Finally, you need to implement one or both of the methods handleEvents() and reportEvents(). These are called when an event/report occurs, and are given the IDs of all events/reports that occurred.

To define a triggered event or triggered report, call createTriggeredEvent() when realizing Instance stage. This returns both an event ID and an index into the Vector of trigger function values. When realizing the appropriate stage, you should calculate the value of the trigger function, then store it in the State by calling

```
state.updEventsByStage(getMySubsystemIndex(), stage)[eventIndex] = value;
```

You then implement handleEvents() to handle the event when it occurs.

3.5.5 Defining new constraints

Another function Subsystems can perform is to define constraints. Once again, doing this directly from a Subsystem is rarely the easiest way. In the previous chapter, we saw an easier way to define constraints using the custom constraint feature of the SimbodyMatterSubsystem.

A constraint equation may be applied to coordinates, velocities, or accelerations. When realizing Instance stage, call allocateQErr() to create a constraint on coordinates, allocateUErr() to create a constraint on velocities, and allocateUDotErr() to create a constraint on accelerations. The actual quantities you will calculate and store in the State are the "constraint errors". Every constraint is defined by an equation of the form $c(\mathbf{d};t,\mathbf{y}) = 0$. You calculate the value of the function $c(\mathbf{d};t,\mathbf{y})$, and Simbody then tries to keep it equal to 0. You set the constraint errors when realizing the appropriate stages by calling updQErr(), updUErr(), and updUDotErr().

It is quite difficult to handle constraints correctly and we highly recommend that you not do this from your own Subsystems but instead take advantage of the Constraint facility that is part of the SimbodyMatterSubsystem, which provides built-in constraints as well as highly flexible custom constraints. See the previous chapter for more information and be sure to take advantage of the Simbody forum at https://simtk.org/home/simbody to discuss with expert users the best approach to solving your problem.