Residual Reduction Algorithm (RRA)

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1 Introduction

2 Reducing Residuals

Throughout this paper, when we use the term "reduce a residual", or "reduce" a DC offset, it really means "try to eliminate" a residual or DC offset. That is, our strategy is to compute parameter alterations that would, in theory, completely eliminate the DC offset of a particular residual. But since the theoretical solutions we compute are not entirely true in practice since there are many factors affecting a particular residual other than just the one parameter we are altering. However, this failure to completely eliminate the DC offset is actually a good thing: we know that there are unmodeled forces in our system (for example, our model has no arms), so we actually do want some small DC offsets to remain for our residuals just to make us feel like we haven't eliminated these unmodeled forces.

To reduce a particular residual R with DC offset $d_R \in \mathbb{R}$ by altering a particular parameter p by an amount Δp, we must compute Δp using the following equation:

$$R_{old} - R_{new} = d_R$$

We require that R_{new} and R_{old} be expressible in terms of inertial parameters and possibly joint variables, and that R_{new} also be expressed in terms of Δp. Then the above equation can be solved for Δp in terms of the inertial parameters and the DC offset d_R. It is easier to see why this equation is true if we look at it this way:

$$R_{new} = R_{old} - d_R$$

Here, R_{old} is the original residual with the DC offset d_R. If we *remove the DC offset* from R_{old}, i.e. if we subtract the DC offset from R_{old}, we get R_{new}.

2.1 Reducing Forward-Backward Rocking

We will reduce the residual MZ by independently altering two parameters: the torso center of mass x-coordinate by an amount Δt_x and the lumbar extension angle by an amount Δl_e.

2.1.1 Altering the Torso Center of Mass

Here we will compute an amount Δt_x by which to alter the x-coordinate of the torso center of mass in order to balance the DC offset of the MZ residual. Let m be the mass of the torso and let $\mathbf{g}$ denote acceleration due to gravity. Let d_{MZ} be the DC offset of the MZ residual. Let $\mathbf{r_0}$ be the moment arm (lever arm) of the torso, which we define to be the vector pointing from the pelvis center of mass to the torso center of mass. Note that $\mathbf{r_0}$ varies as the torso position varies, but its magnitude stays fixed. Then we have that the original value of MZ at any torso position is:

$$MZ_{old} = \mathbf{r_0} \times m\mathbf{g}$$

Let $\mathbf{r_1}$ be the torso moment arm after the center of mass has been displaced in the x direction by Δt_x. Note that $\mathbf{r_1}$ may not have the same magnitude as $\mathbf{r_0}$. Then the new value of MZ is:

$$\begin{aligned} MZ_{new} &= \mathbf{r_1} \times m\mathbf{g} \\ &= (\mathbf{r_0} + (\Delta t_x, 0, 0)) \times m\mathbf{g} \\ &= \mathbf{r_0} \times m\mathbf{g} + (\Delta t_x, 0, 0) \times m\mathbf{g} \end{aligned}$$

The last step is correct since the cross product distributes over addition. Let $\mathbf{d_{MZ}} = (0, 0, d_{MZ})$, i.e. $\mathbf{d_{MZ}}$ is a vector representation of the DC offset. Now we plus the above expressions into the equation $MZ_{old} - MZ_{new} = \mathbf{d_{MZ}}$:

$$\mathbf{r_0} \times m\mathbf{g} - (\mathbf{r_0} \times m\mathbf{g} + (\Delta t_x, 0, 0) \times m\mathbf{g}) = \mathbf{d_{MZ}}$$

The $\mathbf{r_0} \times m\mathbf{g}$ expressions cancel out on both sides, and the value of the remaining cross product is

$$(\Delta t_x, 0, 0) \times m\mathbf{g} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \Delta t_x & 0 & 0 \\ 0 & -mg & 0 \end{vmatrix} = (0, 0, -mg\Delta t_x).$$

So we are left with

$$-(0, 0, -mg\Delta t_x) = (0, 0, d_{MZ})$$

or looking at just the z-coordinates

$$\begin{aligned} mg\Delta t_x &= d_{MZ} \\ \Delta t_x &= \frac{d_{MZ}}{mg}. \end{aligned} \tag{1}$$

So, in order to reduce the DC offset of the MZ residual, i.e. to reduce the average forward-backward rocking motions of a walking model, our computation suggests that we should alter the torso center of mass x-coordinate by an amount d_{MZ}/mg.

2.1.2 Altering the Lumbar Extension Angle

Now we wish to compute an amount Δl_e by which to alter the lumbar extension angle (throughout the entire time interval, not just at the initial time) so that the DC offset for MZ is reduced. We can represent the alteration of the lumbar extension angle with the following geometry: consider the triangle consisting of two vectors $\mathbf{r_0}$ and $\mathbf{r_1}$ with equal length r_0 and with a common starting point with an angle Δl_e between them. Suppose the vectors are oriented so that Δl_e is drawn in a positive sense (counterclockwise) when it is drawn from $\mathbf{r_0}$ to $\mathbf{r_1}$. Let $\mathbf{\Delta l} = \mathbf{r_1} - \mathbf{r_0}$. Assuming Δl_e is small, we can apply an approximation from biomechanics which states that the moment arm (lever arm) of a muscle is equal to $\delta l/\delta\theta$ where δl is the change in length of the muscle when the joint spanned by the muscle rotates by a small angle $\delta\theta$. Applying this approximation to our triangle, we have that

$$r_0 = \Delta l/\Delta l_e$$

$$\Delta l = r_0 \Delta l_e,$$

where $\Delta l = \|\mathbf{\Delta l}\|$. We will show how to compute the (direction of) the vector $\mathbf{\Delta l}$ later. As before, we define $MZ_{new} = \mathbf{r_1} \times m\mathbf{g}$ and $MZ_{old} = \mathbf{r_0} \times m\mathbf{g}$. From the definition of $\mathbf{\Delta l}$, we know that $\mathbf{r_1} = \mathbf{r_0} + \mathbf{\Delta l}$. Plugging into the equation $MZ_{old} - MZ_{new} = \mathbf{d_{MZ}}$, we have

$$\mathbf{r_0} \times m\mathbf{g} - (\mathbf{r_0} \times m\mathbf{g} + \mathbf{\Delta l} \times m\mathbf{g}) = (0, 0, d_{MZ})$$

$$-\mathbf{\Delta l} \times m\mathbf{g} = (0, 0, d_{MZ}).$$

If we write $\mathbf{\Delta l} = (\Delta l_x, \Delta l_y, 0)$, then we have

$$\mathbf{\Delta l} \times m\mathbf{g} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \Delta l_x & \Delta l_y & 0 \\ 0 & -mg & 0 \end{vmatrix} = (0, 0, -mg\Delta l_x).$$

Subsituting into the previous equation, we have

$$mg\Delta l_x = d_{MZ}.$$

Now if we write $\Delta l_x = \Delta l \cos\theta$ where θ is the angle representing the orientation of the vector $\mathbf{\Delta l}$ relative to the positive x-axis (counterclockwise is positive), and since we know the length of the vector is $\Delta l = r_0 \Delta l_e$, we have that $\Delta l_x = r_0 \Delta l_e \cos\theta$, so substituting into the above equation yields

$$mgr_0 \Delta l_e \cos\theta = d_{MZ}$$

$$\Delta l_e = \frac{d_{MZ}}{mgr_0 \cos\theta}.$$

Let α be the angle representing the orientation of $\mathbf{r_0}$, measured in a positive (counterclockwise) sense starting from the positive x-axis. The angle l_e is the orientation of $\mathbf{r_0}$ as measured in a positive (counterclockwise) sense starting from the positive y-axis. So $\alpha = l_e + 90°$. Since we assumed that Δl_e is small, the vector $\mathbf{\Delta l}$ is approximately tangent to the circle with radius r_0 centered at the pelvis center of mass, i.e. we can assume that $\mathbf{\Delta l}$ is just $\mathbf{r_0}$ rotated counterclockwise by $90°$ and scaled. Since we defined θ to be the angle swept counterclockwise from the positive x-axis to $\mathbf{\Delta l}$, then we can assume that $\theta = \alpha + 90° = l_e + 180°$. Hence we have

$$\cos\theta = \cos(l_e + 180°) = \cos l_e \cos 180° - \sin l_e \sin 180° = -\cos l_e$$

so

$$\Delta l_e = -\frac{d_{MZ}}{mgr_0 \cos l_e}. \quad (2)$$

2.2 Reducing Left-Right Rocking

We will reduce the residual MX by independently altering two parameters: the torso center of mass z-coordinate by an amount Δt_z and the lumbar bending angle by an amount Δl_b.

2.2.1 Altering the Torso Center of Mass

Here we will compute an amount Δt_z by which to alter the z-coordinate of the torso center of mass in order to balance the DC offset of the MX residual. Let d_{MX} be the DC offset of the MX residual. The original value of MX at any torso position is:

$$MX_{old} = \mathbf{r_0} \times m\mathbf{g}$$

Let $\mathbf{r_1}$ be the torso moment arm after the center of mass has been displaced in the z direction by Δt_z. Note that $\mathbf{r_1}$ may not have the same magnitude as $\mathbf{r_0}$. Then the new value of MX is:

$$\begin{aligned} MX_{new} &= \mathbf{r_1} \times m\mathbf{g} \\ &= (\mathbf{r_0} + (0, 0, \Delta t_z)) \times m\mathbf{g} \\ &= \mathbf{r_0} \times m\mathbf{g} + (0, 0, \Delta t_z) \times m\mathbf{g} \end{aligned}$$

Let $\mathbf{d_{MX}} = (d_{MX}, 0, 0)$, i.e. $\mathbf{d_{MX}}$ is a vector representation of the DC offset. Now we plus the above expressions into the equation $MX_{old} - MX_{new} = \mathbf{d_{MX}}$:

$$\mathbf{r_0} \times m\mathbf{g} - (\mathbf{r_0} \times m\mathbf{g} + (0, 0, \Delta t_z) \times m\mathbf{g}) = \mathbf{d_{MX}}$$

The $\mathbf{r_0} \times m\mathbf{g}$ expressions cancel out on both sides, and the value of the remaining cross product is

$$(0, 0, \Delta t_z) \times m\mathbf{g} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & \Delta t_z \\ 0 & -mg & 0 \end{vmatrix} = (mg\Delta t_z, 0, 0).$$

So we are left with

$$-(mg\Delta t_z, 0, 0) = (d_{MX}, 0, 0)$$

or looking at just the x-coordinates

$$-mg\Delta t_z = d_{MX}$$
$$\Delta t_z = -\frac{d_{MX}}{mg}. \tag{3}$$

So, in order to reduce the DC offset of the MX residual, i.e. to reduce the average forward-backward rocking motions of a walking model, our computation suggests that we should alter the torso center of mass z-coordinate by an amount $-d_{MX}/mg$.

2.2.2 Altering the Lumbar Bending Angle

Now we wish to compute an amount Δl_b by which to alter the lumbar bending angle (throughout the entire time interval, not just at the initial time) so that the DC offset for MX is reduced. We can represent the alteration of the lumbar bending angle with the following geometry: consider the triangle consisting of two vectors $\mathbf{r_0}$ and $\mathbf{r_1}$ with equal length r_0 and with a common starting point with an angle Δl_b between them. Suppose the vectors are oriented so that Δl_b is drawn in a positive sense (counterclockwise) when it is drawn from $\mathbf{r_0}$ to $\mathbf{r_1}$. Let $\mathbf{\Delta l} = \mathbf{r_1} - \mathbf{r_0}$. Assuming Δl_b is small, we can apply an approximation from biomechanics which states that the moment arm (lever arm) of a muscle is equal to $\delta l/\delta\theta$ where δl is the change in length of the muscle when the joint spanned by the muscle rotates by a small angle $\delta\theta$. Applying this approximation to our triangle, we have that

$$r_0 = \Delta l/\Delta l_b$$
$$\Delta l = r_0 \Delta l_b,$$

where $\Delta l = \|\mathbf{\Delta l}\|$. We will show how to compute the (direction of) the vector $\mathbf{\Delta l}$ later. As before, we define $MX_{new} = \mathbf{r_1} \times m\mathbf{g}$ and $MX_{old} = \mathbf{r_0} \times m\mathbf{g}$. From the definition of $\mathbf{\Delta l}$, we know that $\mathbf{r_1} = \mathbf{r_0} + \mathbf{\Delta l}$. Plugging into the equation $MX_{old} - MX_{new} = \mathbf{d_{MX}}$, we have

$$\mathbf{r_0} \times m\mathbf{g} - (\mathbf{r_0} \times m\mathbf{g} + \mathbf{\Delta l} \times m\mathbf{g}) = (d_{MX}, 0, 0)$$
$$-\mathbf{\Delta l} \times m\mathbf{g} = (d_{MX}, 0, 0).$$

If we write $\mathbf{\Delta l} = (0, \Delta l_y, \Delta l_z)$, then we have

$$\mathbf{\Delta l} \times m\mathbf{g} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & \Delta l_y & \Delta l_z \\ 0 & -mg & 0 \end{vmatrix} = (mg\Delta l_z, 0, 0).$$

Subsituting into the previous equation and extracting just the x-coordinates, we have

$$-mg\Delta l_z = d_{MX}.$$

Now if we write $\Delta l_z = \Delta l \cos\theta$ where θ is the angle representing the orientation of the vector $\mathbf{\Delta l}$ relative to the positive y-axis, and since we know the length of the vector is $\Delta l = r_0 \Delta l_b$, we have that $\Delta l_z = r_0 \Delta l_b \sin\theta$, so substituting into the above equation yields

$$-mgr_0 \Delta l_b \sin\theta = d_{MX}$$
$$\Delta l_b = -\frac{d_{MX}}{mgr_0 \sin\theta}.$$

The angle l_b is the orientation of $\mathbf{r_0}$ as measured in a positive (counterclockwise) sense starting from the positive y-axis. Since we assumed that Δl_b is small, the vector $\mathbf{\Delta l}$ is approximately tangent to the circle with radius r_0 centered at the pelvis center of mass, i.e. we can assume that $\mathbf{\Delta l}$ is just $\mathbf{r_0}$ rotated counterclockwise by $90°$ and scaled. Since we defined θ to be the angle swept counterclockwise from the positive y-axis to $\mathbf{\Delta l}$, then we can assume that $\theta = l_b + 90°$. Hence we have

$$\sin\theta = \sin(l_b + 90°) = \sin l_b \cos 90° + \cos l_b \sin 90° = \cos l_b$$

so

$$\Delta l_b = -\frac{d_{MX}}{mgr_0 \cos l_b}. \tag{4}$$

2.3 Residual Reduction Algorithm (RRA)

The following algorithm will attempt to reduce the MX and MZ DC offsets if it is executed after the inverse kinematics stage of the simulation pipeline and before the CMC stage. We omit implementation details and present only the essential components of RRA here. For our purposes, the inputs needed are:

1. the musculoskeletal model for the subject whose motion is being simulated,
2. the motion file containing the kinematics and ground reaction data for the subject at a finite set of discrete time instants $t_1, t_2, \ldots, t_N$,
3. whether torso x or lumbar extension will be used to reduce the MX DC offset, and
4. whether torso z or lumbar bending will be used to reduce the MZ DC offset.

The first pass of RRA consists of the following steps.

1. Run CMC once on the subject model using the given motion data.
2. Compute the torso x correction amount Δt_x using equation 1.
3. Compute the torso z correction amount Δt_z using equation 3.
4. Create two arrays Δl_e and Δl_b that can each hold N real numbers.
5. For each $i = 1, \ldots, N$, compute the lumbar extension correction amount $\Delta l_e[i]$ at the ith time step in the motion data using equation 2 and the corresponding value $l_e[i]$ of the model's lumbar extension angle at that time step.
6. For each $i = 1, \ldots, N$, compute the lumbar bending correction amount $\Delta l_b[i]$ at the ith time step in the motion data using equation 4 and the corresponding value $l_b[i]$ of the model's lumbar bending angle at that time step.
7. Write the numbers Δt_x and Δt_z and the arrays Δl_e and Δl_b to a file. If $|\Delta t_x| > 0.1$, set Δt_x to zero before writing it to the file. Do the same for Δt_z. If for any i, $|\Delta l_e[i]| > 10°$, then set every entry in the array Δl_e to zero before writing its entries to the file. Do the same for the array Δl_b. The next pass of RRA will automatically apply the values in this file as corrections to the input model and input motion data. So if any correction amount is listed as zero in the file, the result of applying that correction amount to the input model or motion data is the equivalent of making no correction at all. For instance, if $\Delta t_x = 0$, then adding Δt_x to the original value of t_x in the second pass of RRA is the same as making no change to the original value of t_x.
8. If every correction exceeded the threshold amounts (i.e. if every number in the entire file written in the previous step was zero), then exit with a message saying that this subject data cannot be corrected by RRA.

The second pass of RRA consists of the following steps.

1. Read Δt_x, Δt_z, Δl_e, and Δl_b from the file created in the first pass of RRA.
2. If the user chose t_x as the parameter to alter in order to reduce the MX DC offset, add Δt_x to the model's torso x coordinate.
3. If the user chose t_z as the parameter to alter in order to reduce the MZ DC offset, add Δt_z to the model's torso z coordinate.
4. If the user chose l_e as the parameter to alter in order to reduce the MX DC offset, then for each $i = 1, \ldots, N$, add $\Delta l_e[i]$ to $l_e[i]$.
5. If the user chose l_b as the parameter to alter in order to reduce the MZ DC offset, then for each $i = 1, \ldots, N$, add $\Delta l_b[i]$ to $l_b[i]$.
6. Run CMC on the subject model using the given motion data.

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