

We have (boldface indicates vector)

$$\mathbf{F}_{grav} + \mathbf{F}_{grf} + \mathbf{F}_{residual} = m\mathbf{a}$$

and  $\mathbf{F}_{grav} = m\mathbf{g}$ , so that

$$\mathbf{F}_{residual} = m(\mathbf{a} - \mathbf{g}) - \mathbf{F}_{grf}$$

and focusing on the y component ( $g_y = -9.80665m/s^2$ )

$$F_{residual,y} = m(a_y - g_y) - F_{grf,y}$$

We can compute the means of these functions across the time interval of interest:

$$\bar{F}_{residual,y} = m(\bar{a}_y - g_y) - \bar{F}_{grf,y}$$

and we want to find an adjusted mass  $m^*$  which will make the FY mean be zero, i.e.

$$0 = m^*(\bar{a}_y - g_y) - \bar{F}_{grf,y}$$

Subtracting the two equations and rearranging, we get

$$m^* = m + \bar{F}_{residual,y}/(g_y - \bar{a}_y)$$

Compare this with our current approach of taking

$$m^* = m + \bar{F}_{residual,y}/g$$

If the acceleration averages out to zero (i.e.  $\bar{a}_y = 0$ ) then these will give the same answer... but if that's not the case, the formula including the  $\bar{a}_y$  term will be more accurate (will get the new  $\bar{F}_{residual,y}$  to be zero).