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A Unified Method for Calculating the Center of Pressure during Wheelchair Propulsion

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Abstract—The measurement of the center of pressure (COP) has been and continues to be a successful tool for gait analysis. The definition of a similar COP for wheelchair propulsion, however, is not straightforward. Previously, a COP definition similar to that used in force plate analysis had been proposed. Unfortunately, this solution has the disadvantage of requiring a separate COP definition for each plane of analysis. A definition of the generalized center of pressure (GCOP) which is consistent in all planes of analysis is derived here. This definition is based on the placement of a force-moment system, equivalent to the force-moment system at the hub, on a line in space where the moment vector (wrench moment) is parallel to the force vector. The parallel force-moment system is then intersected with three planes defined by anatomical landmarks on the hand. Data were collected using eight subjects at propulsion speeds of 1.34 m/s and 2.24 m/s (1.34 m/s only for subject 1, 0.894 m/s and 1.79 m/s for subject 8). Each subject propelled a wheelchair instrumented with a SMART^{Wheel}. A PEAK 5 video system was used to determine the position of anatomical markers attached to each subject's upper extremity. The GCOP in the transverse plane of the wrist formed clusters for all subject's except subject 2 at 1.34 m/s. The clustering of the GCOP indicates that the line of action for the force applied by the hand is approximately perpendicular to the transverse plane through the wrist. When comparing the magnitude of the moment vector part of the wrench with the moment of the force vector of the wrench about the hub, the wrench moment is approximately an order of magnitude smaller. This indicates that the role of the wrist for wheelchair propulsion is primarily to stabilize the force applied by the arm and shoulder. © *1998 Biomedical Engineering Society.* [S0090-6964(98)00502-5]

Keywords—SMART^{Wheel}, Kinematics, Kinetics, Rehabilitation, Biomechanics, Generalized center of pressure.

INTRODUCTION

Currently there are many types of measurements used for the study of wheelchair propulsion biomechanics, many of which are analogous to those used in the analysis of ambulatory gait.[7,16,17,23–25] Commonly used measurements for gait analysis include the forces and moments at the hand, the forces and moments at the joints, several time intervals, and the center of pressure (COP). The focus of this article is the development of a new COP measurement for wheelchair propulsion much like the two-dimensional COP measurement used for the study of ambulation.[10,14,24] Recently, Cooper *et al.*[7] introduced a definition of the COP for wheelchair propulsion. With Cooper's definition, multiple COPs are determined, one for each prescribed plane of analysis. When used for the analysis of wheelchair propulsion, multiple planes are necessary due to the ability of the hand to grab the pushrim and apply a moment to the pushrim not related to the applied force. In comparison, the calculation of the ambulatory COP on a force plate is a simplified case as the only plane used is the horizontal plane defined by the force plate surface.[26] The restrictions of a multiple COP definition have not been limiting for ambulation studies because the foot is incapable of grabbing a force plate. Another difference between a COP defined for wheelchair propulsion and a COP defined for ambulation is that a wheelchair propulsion COP does not have to be within the confines of the hand. This is also due to the hand's ability to grab the rim. Unfortunately, the direct calculation of a single point COP for three-dimensional Cartesian space is not possible for an arbitrarily applied moment and force combination.[7] Therefore, a generalized solution is sought where the definition of the generalized center of pressure (GCOP) is unified for all planes.

METHODS

Single Point COP

As described in Cooper *et al.*,[7] the COP is a point where the force applied by the hand onto the pushrim

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produces precisely the moment measured at the hub of the pushrim. This was calculated from force and moment data measured with the SMART$^{\text{Wheel}}$.[6,7] The SMART$^{\text{Wheel}}$ measures the resultant force through and the moment about the hub of the pushrim. This same concept is used in the calculation of the ambulatory COP using a force plate. For the calculation of the COP on a force plate, the moments perpendicular to the horizontal plane (M_z) and the forces parallel to the horizontal (F_x and F_y) plane are ignored. This results in a simple definition of the COP as given in Eq. (1). F_z is the vertical force and M_x and M_y are the horizontal moments as measured relative to the center of the force plate:

$$\mathbf{COP} = -\frac{M_y}{F_z}\,\mathbf{i} + \frac{M_x}{F_z}\,\mathbf{j}. \tag{1}$$

When utilizing the SMART$^{\text{Wheel}}$, planes other than the horizontal plane can be used for analysis. By using planes other than the horizontal plane, different components of the force and moment vectors measured at the hub will contribute to the COP location. Unlike the ambulation case, there is no identifiable ''most significant'' plane for wheelchair propulsion analysis. Therefore, it is desirable to find a generalized center of pressure (GCOP) with a unified definition for all planes.

If the force and moment vectors measured at the hub are perpendicular, then there is a line where the moment of an equivalent force-moment system will be equal to zero. Any intersection of this line with a plane would be directly analogous to the two-dimensional COP through for the hand. For all other force and moment combinations measured at the hub, there is no position where an equivalent force-moment system can be placed such that the moment has zero magnitude.

The placement of an equivalent force-moment system at a location where the resulting moment is parallel to the force provides the basis for the GCOP definition. This moment, parallel to the force vector, is called a wrench moment.[3] The wrench moment is significant in that this moment must have been produced by the wrist and cannot be accounted for by a net force applied to the pushrim alone. This does not mean that this is the only moment generated by the hand-wrist system, but it does give an indication of wrist musculature activity. Furthermore, if the wrench moment is small in comparison to the magnitude of the measured moment at the hub, then a GCOP would be approximately analogous to a two-dimensional COP.

A hand coordinate system, based on anatomical markers of the hand, is necessary because the eventual calculation of a GCOP will be the intersection of the parallel force-moment system with a plane defined by the hand. Figure 1 shows the locations of the four physical markers (labeled 1, 2, 3, and 4) and the basis vectors of the hand coordinate system. Marker 4 was included for use with other anatomical measurements not considered in this study. The hand coordinate system is based only on the locations of markers 1, 2, and 3. The origin is defined as the midpoint between markers 1 and 2. The basis vector $\mathbf{j}'$ is chosen to be the normalized vector $\mathbf{R}_{12}$ (vector from marker 1 to marker 2).

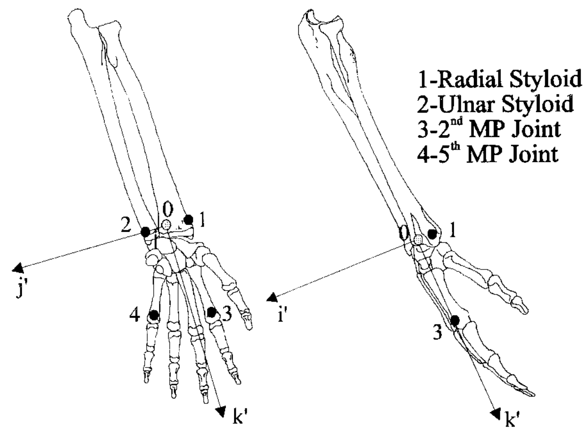

FIGURE 1. Anatomical marker positions and definition of the hand coordinate system. The hand coordinate system is based on the positions of markers 1, 2, and 3 only.

Equation (2) describes vector $\mathbf{k}'$. This vector is the basis vector of $\mathbf{R}_{13}$ made orthogonal to $\mathbf{j}'$ by subtracting off the component of $\mathbf{R}_{13}$ that is parallel to $\mathbf{j}'$. This procedure is similar to the Gram–Shmidt orthogonalization procedure[20,21] which is used to produce an orthogonal matrix from a matrix of linearly independent vectors:

$$\mathbf{k}' = \mathbf{R}_{13} - \frac{\mathbf{R}_{12}\times\mathbf{R}_{13}}{|\mathbf{R}_{12}|^2}\,\mathbf{R}_{12}. \tag{2}$$

$\mathbf{i}'$ is defined to be the cross product of $\mathbf{j}'$ and $\mathbf{k}'$ [Eq. (3)]. Given this definition $\mathbf{i}'$ will be normal to the plane formed by markers 1, 2, and 3, and will point toward the dorsum of the hand:

$$\mathbf{i}' = \mathbf{j}'\times\mathbf{k}'. \tag{3}$$

Using Eq. (4), any point $\mathbf{R}$ (R_x, R_y, R_z) in the laboratory coordinate system can be described in the hand coordinate system where x_0, y_0, z_0 is the origin of the hand coordinate system. The laboratory coordinate system is a right-hand Cartesian coordinate system centered at the hub with the x axis ($\mathbf{i}$) forward, the y axis ($\mathbf{j}$) up, and the z axis ($\mathbf{k}$) to the wheelchair rider's right. The following matrix represents the three vectors $\mathbf{i}'$, $\mathbf{j}'$, $\mathbf{k}'$, placed on the rows:

$$\begin{bmatrix} R'_x \\ R'_y \\ R'_z \end{bmatrix} = \begin{bmatrix} i'_1 & i'_2 & i'_3 \\ j'_1 & j'_2 & j'_3 \\ k'_1 & k'_2 & k'_3 \end{bmatrix} \begin{bmatrix} R_x - x_0 \\ R_y - y_0 \\ R_z - z_0 \end{bmatrix}. \quad (4)$$

There is no single position in space that is directly analogous to the two-dimensional COP.[7] A line of positions can be determined, however, such that the force applied through the pushrim to the center of the hub (**F**) accounts for the entire moment component ($\mathbf{M}_R$) perpendicular to **F**. The remaining wrench moment ($\mathbf{M}_W$) is therefore parallel to **F**. In this manner, the moment (**M**) measured at the center of the hub by the SMART$^{\text{Wheel}}$ can be considered to consists of two components as shown in Eq. (5). A GCOP is then defined by the intersection of this line with a particular plane. To determine the location of this line, a point ($\tilde{\mathbf{R}}$) on the line is found closest to the hub. The magnitudes of $\mathbf{M}_R$ and $\tilde{\mathbf{R}}$ can be found by using Eqs. (6) and (7):

$$\mathbf{M} = \mathbf{M}_R + \mathbf{M}_W . \quad (5)$$

Equation (6) is based on the property of the cross product magnitude and is the product of the magnitudes of the individual vectors multiplied by the sine of the angle ($\theta_{F,M}$) between them:

$$\|\mathbf{M}_R\| = \frac{\|\mathbf{F}\|\|\mathbf{M}\|\sin(\theta_{F,M})}{\|\mathbf{F}\|} = \frac{\|\mathbf{F}\times\mathbf{M}\|}{\|\mathbf{F}\|}, \quad (6)$$

$$\|\tilde{\mathbf{R}}\| = \frac{\|\mathbf{M}_R\|}{\|\mathbf{F}\|} = \frac{\|\mathbf{F}\times\mathbf{M}\|}{\|\mathbf{F}\|^2}. \quad (7)$$

$\tilde{\mathbf{R}}$ must be perpendicular to **F** and $\mathbf{M}_R$ by definition. The direction of $\tilde{\mathbf{R}}$ must then be along the direction of $\mathbf{M}_R$ cross **F**. But, the direction (and magnitude) of $\mathbf{M}_R$ cross **F** is equivalent to **M** cross **F** [Eq. (8)]. Multiplication of Eqs. (7) and (8) leads to the position $\tilde{\mathbf{R}}$ [Eq. (9)]:

$$\frac{\tilde{\mathbf{R}}}{\|\tilde{\mathbf{R}}\|} = \frac{\mathbf{F}\times\mathbf{M}_R}{\|\mathbf{F}\times\mathbf{M}_R\|} = \frac{\mathbf{F}\times\mathbf{M}}{\|\mathbf{F}\times\mathbf{M}\|}, \quad (8)$$

$$\tilde{\mathbf{R}} = \frac{\mathbf{F}\times\mathbf{M}}{\|\mathbf{F}\|^2}. \quad (9)$$

Since the hand is applying the force and the moment to the pushrim, it is natural to use a plane defined relative to the hand for a GCOP definition. Using Eq. (4) $\tilde{\mathbf{R}}$ can be expressed in the hand coordinate system as $\tilde{\mathbf{R}}'$. Equation (10) is the point-vector definition of the line formed by the parallel force-moment system. In Eq. (10), $\tilde{\mathbf{F}}'$ is the applied force measured at the hub in the hand coordinate system. The solution of Eq. (10) is accomplished by substitution by setting one component of the **GCOP** components to zero, determining σ, and then finding the other two components of a **GCOP** vector:

$$\frac{\mathbf{F}'}{\|\mathbf{F}'\|}\sigma + \tilde{\mathbf{R}}' = \begin{bmatrix} \mathrm{GCOP}_x \\ \mathrm{GCOP}_y \\ \mathrm{GCOP}_z \end{bmatrix}. \quad (10)$$

Experimental Protocol

The kinetic data were collected with a SMART$^{\text{Wheel}}$.[2,7,22] The SMART$^{\text{Wheel}}$ is a pushrim force and moment sensor that was designed, fabricated, and calibrated at the Human Engineering Research Laboratories. Eight volunteers gave written informed consent for this experimental protocol. All of the subjects were experienced wheelchair users with a disability. Each subject had had a traumatic spinal cord injury, except subject 1 who had cerebral palsy. The subjects wore black fingerless gloves with reflective markers attached. The markers consisted of 6 mm Styrofoam balls covered with highly reflective tape. Most of the light incident on the reflective markers is reflected towards the source. The reflective markers were placed over the radial styloid, ulnar styloid, second metacarpophalangeal joint, and the fifth metacarpophalangeal joint.[13] These positions correspond to markers numbered 1–4, respectively (Fig. 1). To establish a reference, a 12 mm marker was placed at the hub of the SMART$^{\text{Wheel}}$. Each of the subjects propelled the wheelchair at two different speeds separated by a 5 min rest period. Subjects 2–7, propelled first at 1.34 m/s and then at 2.24 m/s. Subject 1 was unable to reach 2.24 m/s and this test was not performed. Subject 8 propelled at 0.894 and 1.79 m/s. Data were collected for the last 15 s of each test period. All of the subjects propelled a Quickie GPV wheelchair with a seat depth of 0.41 m and a seat width of 0.41 m. The size and type of this wheelchair is similar to the subjects' own wheelchairs when measured according to ISO standard 7176-7.[8] The test wheelchair was mounted on a dynamometer with the entire load due to friction.

For the purposes of this protocol, the angular rotation of the SMART$^{\text{Wheel}}$ was determined using the video data. The angular rotation is given by

$$\theta = \arctan\left(\frac{y_{\mathrm{rim}} - y_{\mathrm{hub}}}{x_{\mathrm{rim}} - x_{\mathrm{hub}}}\right). \quad (11)$$

The ATAN2 function of MatLab[11] was used to determine the inverse tangent because the angle is returned in the proper quadrant given the signs of the numerator and denominator. The MatLab[11] programming environment

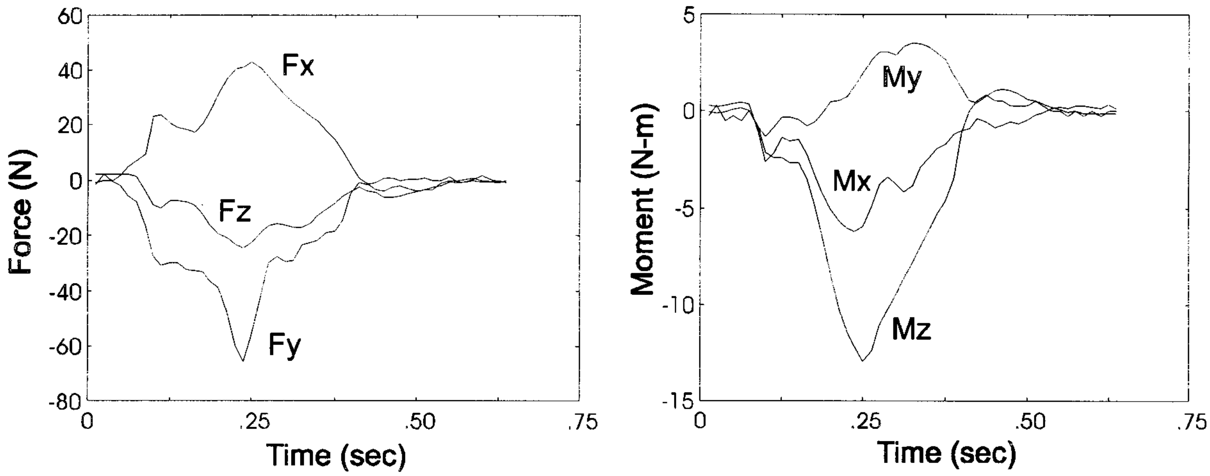

FIGURE 2. Typical forces and moment time series for one propulsive stroke. The data were parsed manually. Subject 1 at 1.34 m/s.

was used for the implementation of these algorithms.

All of the data were collected from each subject's right side. The positions of the reflective markers were recorded through the use of a PEAK5 (PEAK5 Systems Technology, Inc.) video analysis system which uses three cameras. The first camera was placed perpendicular to the sagittal plane, in-line with the rear wheels. The second camera was placed at a 45° angle, between the sagittal and frontal planes. The third camera was placed at a 45° angle, between the frontal and horizontal planes looking down at the subject. The video data were sampled at 60 frames/s. The pushrim forces and moments were measured using the SMART$^{\text{Wheel}}$.[2,7,22] The SMART$^{\text{Wheel}}$ was calibrated before and after all tests were performed. The pushrim force and moment data were sampled at 240 Hz per channel and filtered with a 3rd order, zero-phase, Butterworth digital filter with a cutoff frequency of 40 Hz. The time base for the video data was increased to 240 Hz by means of linear interpolation. Five strokes were analyzed for each subject. The strokes were parsed manually by the inspection of all moments and forces plotted in the time domain. Synchronization between both SMART$^{\text{Wheel}}$ and the PEAK5 system was achieved with an electronic pulse at the beginning of data collection. The data were reviewed following each test to ensure proper collection.

RESULTS

Figure 2 is a set of time series graphs for the moment and force applied by the hand to the pushrim as referenced to the hub (subject 1, 1.34 m/s). Figures 3–9 show the location of the GCOP as obtained by the intersection of the line given in Eq. (10) with the $\mathbf{i}'-\mathbf{j}'$ or the $\mathbf{j}'-\mathbf{k}'$ planes of the hand coordinate system. The deviation in the distance between the markers is due to the movement of the glove over the kind and movement of the skin as well. This effect is commonly referred to as skin artifact. The origin of the hand coordinate system is the midpoint between markers 1 and 2. A vector from marker 1 to marker 2 is always parallel to the $\mathbf{j}'$ axis while the $\mathbf{k}'$ axis is the direction of the vector from marker 1 to marker 3, but is not necessarily parallel to it [Eq. (2)].

Figures 3–6 are from subjects 3 and 7 at both 1.34 and 2.24 m/s. In Figs. 3–6, the data tends to cluster within approximately 10 cm of the local hand origin with

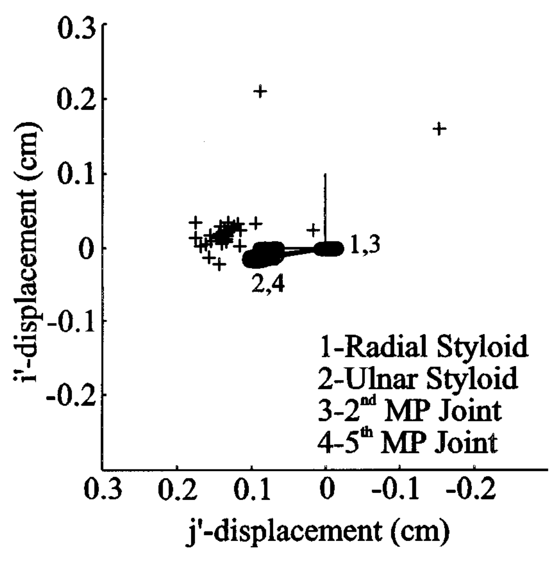

FIGURE 3. Cluster of the GCOP (+) lateral and dorsal to the ulnar styloid. Subject 3, 1.34 m/s.

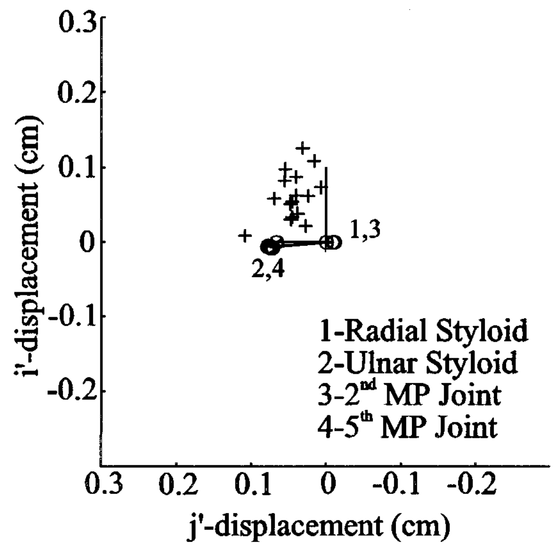

FIGURE 4. GCOP (+) cluster dorsal to the radial styloid and ulnar styloid. Subject 3, 2.24 m/s.

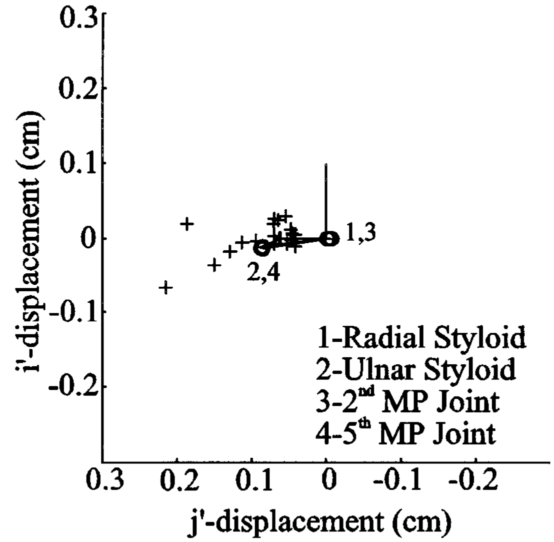

FIGURE 6. GCOP (+) cluster crossing through the hand plane toward the ulnar styloid and lateral to the ulnar styloid toward the end of the propulsion stroke. Subject 7, 2.24 m/s.

the exception of data from the extreme beginning (1–3 data points at 240 Hz) and ending (1–3 data points) of the propulsion stroke where the variability of the GCOP is high. An exception to the cluster nature of the GCOP was noted for subject 2 at 1.34 m/s as shown in Fig. 7. Subject 2 consistently produced this pattern for all of the slower speed strokes. The GCOP data for subject 2 at 2.24 m/s, however, appears to be similar to that of the other subjects. Figure 9 illustrates the alternate use of the $\mathbf{j}'-\mathbf{k}'$ plane intersection instead of the $\mathbf{i}'-\mathbf{j}'$ plane intersection. The data in this plane (the $\mathbf{j}'-\mathbf{k}'$ plane), as well as the for the $\mathbf{i}'-\mathbf{k}'$ intersection plane, show a progression of the GCOP as compared to the clustering nature noted in the $\mathbf{i}'-\mathbf{j}'$ plane intersection.

Figure 10 is a time series plot of the moments about the hub calculated from the position of the GCOP and the force vector acting through it. The data that were used in Fig. 10 are the same as were used for Fig. 2. Figure 11 illustrates the wrench moment time sequence for this same data set. The mean-square difference is

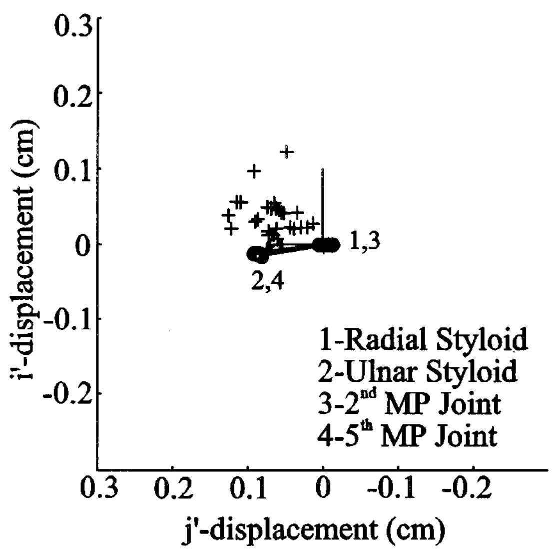

FIGURE 5. GCOP (+) cluster dorsal to radial styloid and ulnar styloid. Subject 7, 1.34 m/s.

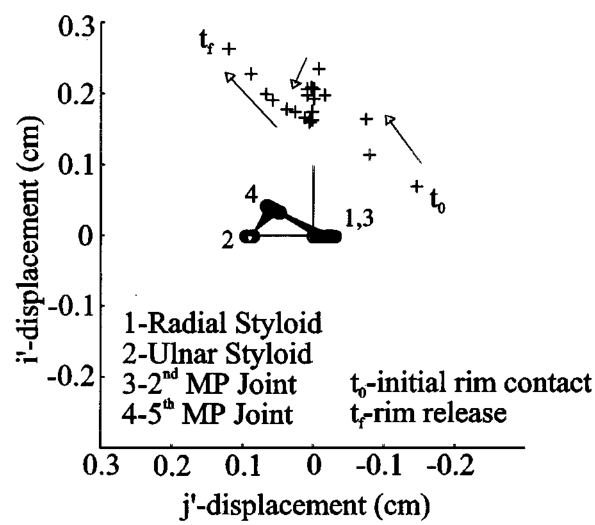

FIGURE 7. This shows an unusual scatter of data. The arrow indicates the direction of the proceeding GCOP (+) with respect to time. Subject 2, 1.34 m/s.

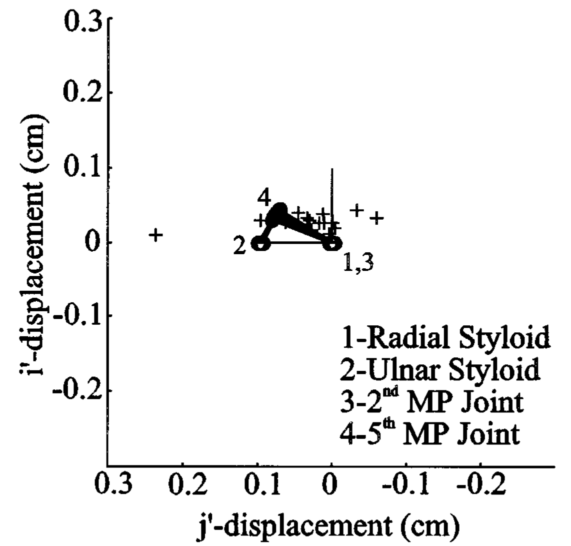

FIGURE 8. Unlike Fig. 7, subject 2 at 2.24 m/s shows a similar GCOP (+) cluster to that of all other subjects at both speeds.

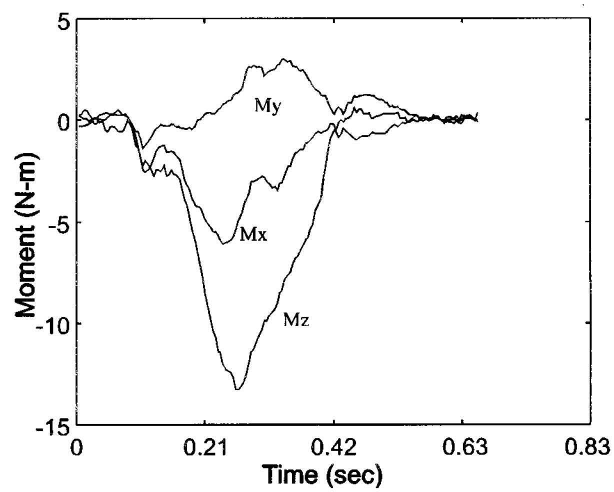

FIGURE 10. Moments about the hub due to a force offset from wrist. These moments are due the cross product of the applied force through the GCOP and the distance to the GCOP from the hub in the laboratory coordinate system. Subject 1, 1.34 m/s.

used in Table 1 to compare the magnitude of the moment perpendicular to the applied force vector with the magnitude of the moment measured at the hub (MSDP). The magnitude of the wrench moment was compared to the magnitude of the hub moment in a similar fashion (MSDW). The data in Table 1 are from the first propulsive stroke of each trial. In all cases, the mean-square difference was much greater for the comparison involving the wrench moment magnitude than for the magnitude of the moment perpendicular to the applied force vector.

DISCUSSION

The beginning and ending values for GCOP or COP data over a single propulsion stroke or over a step on a

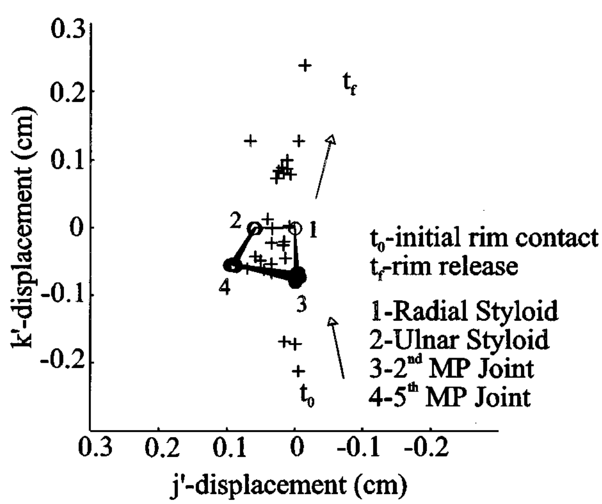

FIGURE 9. This is the GCOP (+) using the intersection of the line of solutions with the j′−k′ plane instead of the i′−j′ plane. This data shows considerably greater scatter which is typical of this intersection. The arrows indicate the proceeding GCOP (+) with respect to time. Subject 1, 1.34 m/s.

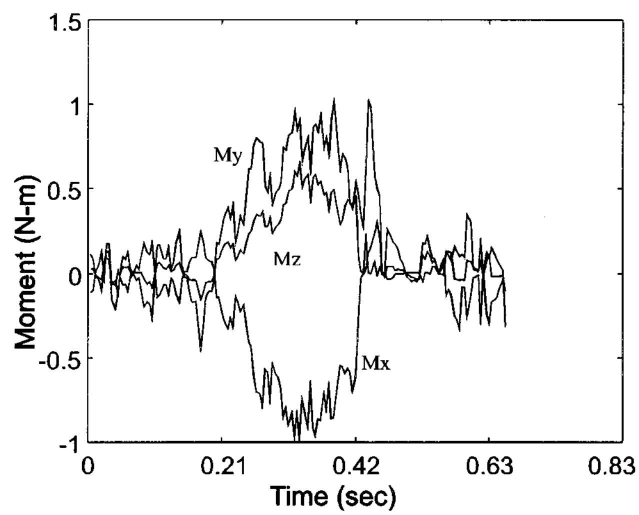

FIGURE 11. Wrench moment magnitude parallel to *F*. These moments are the difference between the measured moment at the hub and the moment data calculated from the force applied by the hand through the GCOP. This data represent moments which must have been generated by the wrist and is not the result of a lever action.

TABLE 1. Mean-square difference between the perpendicular moment magnitude and the moment magnitude at the hub (MSDP); and the mean-square difference between the wrench moment and the moment magnitude at the hub (MSDW).

Subject	Speed	MSDP	MSDW	*n*
1	1.34	0.0103	52.5	94
	2.24	------	------	---
2	1.34	0.0324	116	84
	2.24	0.0244	150	56
3	1.34	0.0904	86.2	87
	2.24	0.0069	113	59
4	1.34	0.0043	12.8	91
	2.24	0.0572	13.3	59
5	1.34	0.0031	13.9	108
	2.24	0.0039	37.0	64
6	1.34	0.0046	18.3	118
	2.24	0.0095	51.6	81
7	1.34	0.1144	42.6	77
	2.24	0.0291	56.8	56
8	0.894	0.1389	170	136
	1.79	0.1601	306	91

force plate are not considered to be accurate due to the small magnitudes of the measured forces and moments.[4,12] The reason for this problem is twofold. First, when the magnitude of either the applied force or moment becomes small, the signal to noise ratio is reduced. This is compounded by the fact that in Eqs. (1) and (9), the force appears in the denominator. This effect was noticed with the GCOP as well, but manual parsing of the data over a propulsive stroke proved to be an effective solution.

Neither the $\mathbf{j}'-\mathbf{k}'$ nor the $\mathbf{i}'-\mathbf{k}'$ intersection plane appeared to be useful in comparison to the $\mathbf{i}'-\mathbf{j}'$ plane. The $\mathbf{j}'-\mathbf{k}'$ and the $\mathbf{i}'-\mathbf{k}'$ intersection planes produced a progression of GCOP data points. This progression is likely because the parallel force-moment combination passes through these planes at a shallow angle. Therefore, a small change in the location of the line of solution would lead to a large change in the point of intersection. This indicates that the parallel force-moment system lies approximately perpendicular to the $\mathbf{i}'-\mathbf{j}'$ plane. This is not the same as saying that the line of action for the applied force is parallel to the radius or ulna because the hand rotates relative to the radius and ulna throughout the propulsive stroke.

Wheelchair use has been linked to an increase in the incidence of Carpal Tunnel Syndrome[1,8] as well as to other neuropathies.[4,5,9,15,18,19,28] Two primary risk factors have been identified: (1) large moments at the wrist and (2) the repetitive nature of the propulsion motions.[27] The wrist moment is the moment of an equivalent force-moment system located at the wrist articulation center (the hand as a freebody separated at the wrist). In analyzing the wrist moment, it is necessary to account for the individual's anthropometric dimensions. The GCOP is calculated from both the wrist moment and the applied force. A larger individual may generate a large moment at the wrist, but the distance from the GCOP to the wrist articulation center may actually be the same or similar to that of a smaller individual. Furthermore, an activity that produces a low wrist moment may have a GCOP located further from the wrist articulation center than an activity with a higher wrist moment. The GCOP location could potentially be used as a measure to help further optimize the propulsion stroke for the minimization of repetitive strain injury. Since the GCOP can be visualized as the line through which the applied force is directed, the GCOP may prove to provide a highly intuitive feedback mechanism if graphed in real time with a superimposed image of the subject's hand.

It is evident from Figs. 10 and 11 that the majority of the moment measured at the hub is due to the applied force and not the wrench moment. The absolute peak magnitude of the wrench moment component in Fig. 11 (M_x) is less than 1 N m. The absolute peak component magnitude of the moment due to the applied force through the GCOP (M_z) is greater than 13 N m. Veeger *et al.*[24] used a two-dimensional method for determining the moment attributed to the hand. Their results indicate that the hand contributes approximately 31% of M_z. However, the mean-square difference data presented in Table 1 indicates that the moment measured at the hub and the moment generated by the force applied by the hand are nearly equal. The discrepancy between Veeger's work and the research presented here may be attributable to Veeger's use of a two-dimensional solution. This solution fails to consider that the applied force by the hand may not be in the plane of the wheel.

It is possible to consider that the wrist moment is composed of two components, an impedance component and an active component. The impedance component of the wrist moment is the moment that is necessary to resist the application of force through the GCOP by the upper extremity. The active component is the propulsive effort provided solely by the musculature of the wrist. The results presented here indicate that the wrist does exert an impedance moment because the GCOP deviates from the wrist articulation center. The results do not indicate that the wrist plays an active role in wheelchair propulsion. This is because nearly all of the moment measured at the hub can be attributed to a force acting through the GCOP. If the wrist musculature did play an active role in wheelchair propulsion, it is likely that a significant wrench moment would have been detected.

The parallel force-moment system is not the only force-moment system that could be used for the analysis of wheelchair propulsion. It could be argued that any equivalent force-moment system could be considered. Given that the wrench moment of a parallel force-moment system is small, a single force acting through the GCOP is the simplest model which accounts for the measured forces and moments. However, it may be revealed that the very small net wrench moment (>1.0 N m) may still be important in the study of wrist injury. Further studies will be needed to confirm the implication of these results.

NOMENCLATURE

$\mathbf{i},\mathbf{j},\mathbf{k}$	unit vectors in the laboratory coordinate system
$\mathbf{i}',\mathbf{j}',\mathbf{k}'$	unit vectors in the local hand coordinate system
$\mathbf{R}'$	position vector in the local hand coordinate system
$\tilde{\mathbf{R}}$	a point along the line of solutions for the center of pressure; used to find the line of action of the applied force
$\mathbf{F}'$	force vector applied to the hub measured by the SMART$^{\text{Wheel}}$ in the hand coordinate system
θ	angular position of the SMART$^{\text{Wheel}}$; when mounted on the right side of a wheelchair, the angular rotation is negative when the wheelchair is propelled forward
$\theta_{F,M}$	angle between the vectors **F** and **M**
σ	variable parameter used for the point-vector definition of a line
COP	center of pressure
F	force vector applied to the hub measured by the SMART$^{\text{Wheel}}$; the applied force by the hand is assumed to be equal
GCOP	generalized center of pressure
GCOP	position vector to the generalized center of pressure in the hand coordinate system
M	moment vector applied to the hub measured by the SMART$^{\text{Wheel}}$
$\mathbf{M}_R$	moment component in the laboratory coordinate system perpendicular to **F**
MSDP	mean-square difference between the magnitude of the moment perpendicular to the applied force vector and the magnitude of the moment measured at the hub
MSD	mean-square-difference between the wrench moment magnitude and the magnitude of the moment measured at the hub
$\mathbf{M}_W$	wrench moment; moment component parallel to **F**
R	generic position vector

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