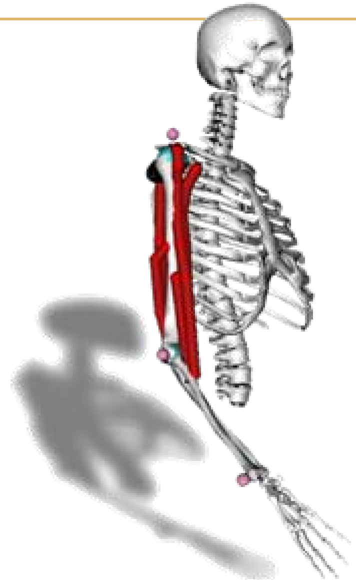
Modeling “Biological” Joints in Simbody™

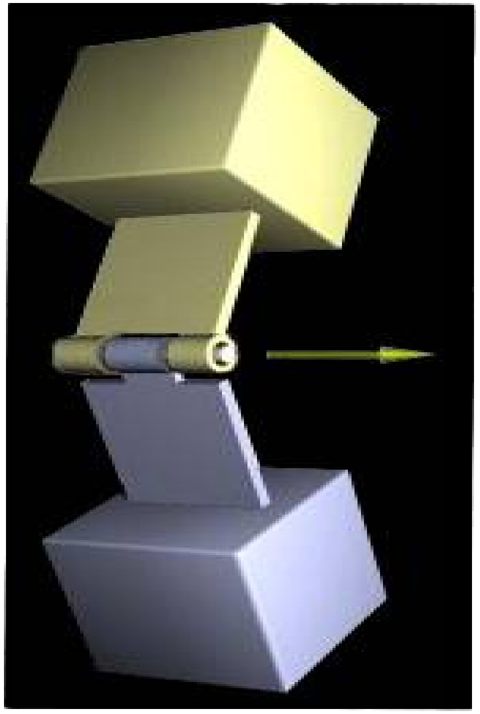
Ajay Seth

Simbios



Modeling Biological Joints

Hinge (pin joint)



Finger

Elbow

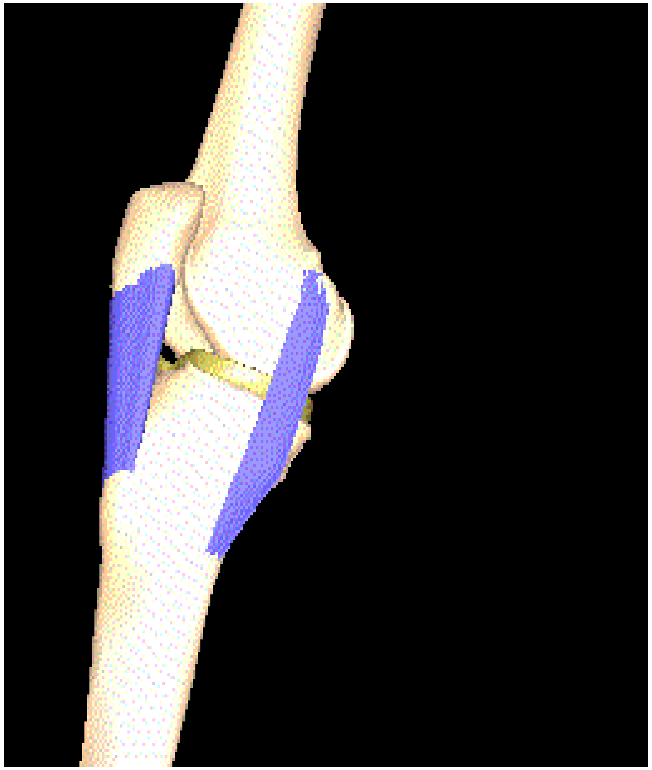
Ideal Rotation

Bones Rotate + Translate

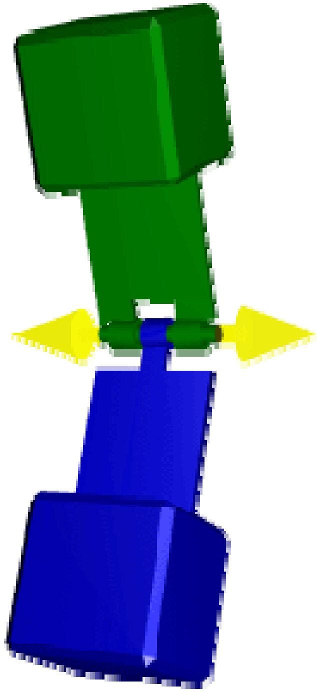
Implementing Biological Joints

- Standard Approach (in other codes):
  - Include coordinates to describe translations
  - Add constraints to prescribe translations in terms of rotation
  - Slower than ideal (pin, ball-socket) joint
- Simbody:
  - Motion described by one coordinate
  - No constraints
  - Similar performance to ideal joint

The Human Knee

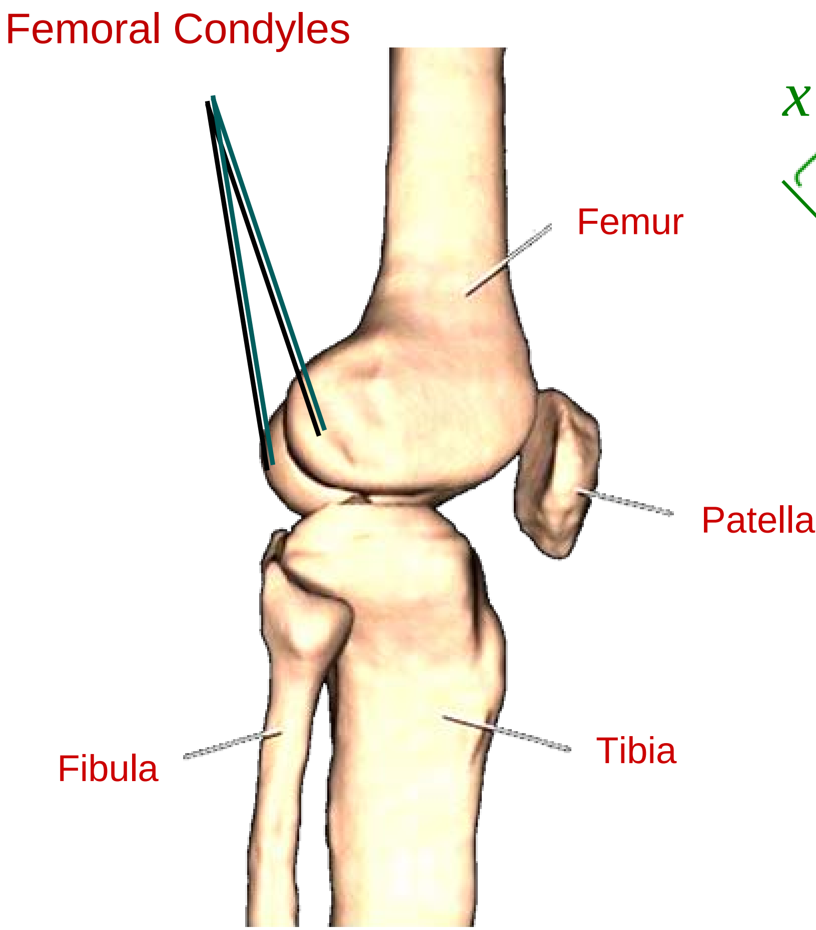


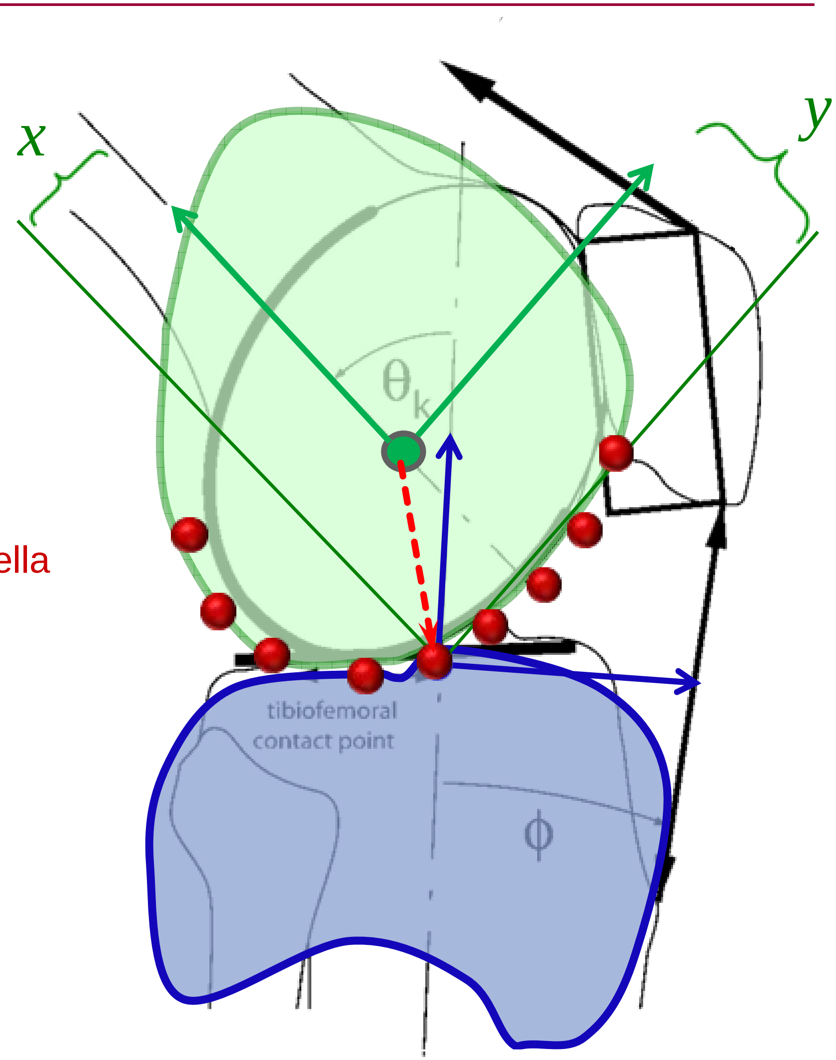
Musculographics, Inc.



Ryan Blumenthal

Sagittal Plane Knee Kinematics





Yamaguchi & Zajac 1989

A Knee Mobilizer

Cadaver experiments:
measure translations (x, y)
of tibia w.r.t. femur.

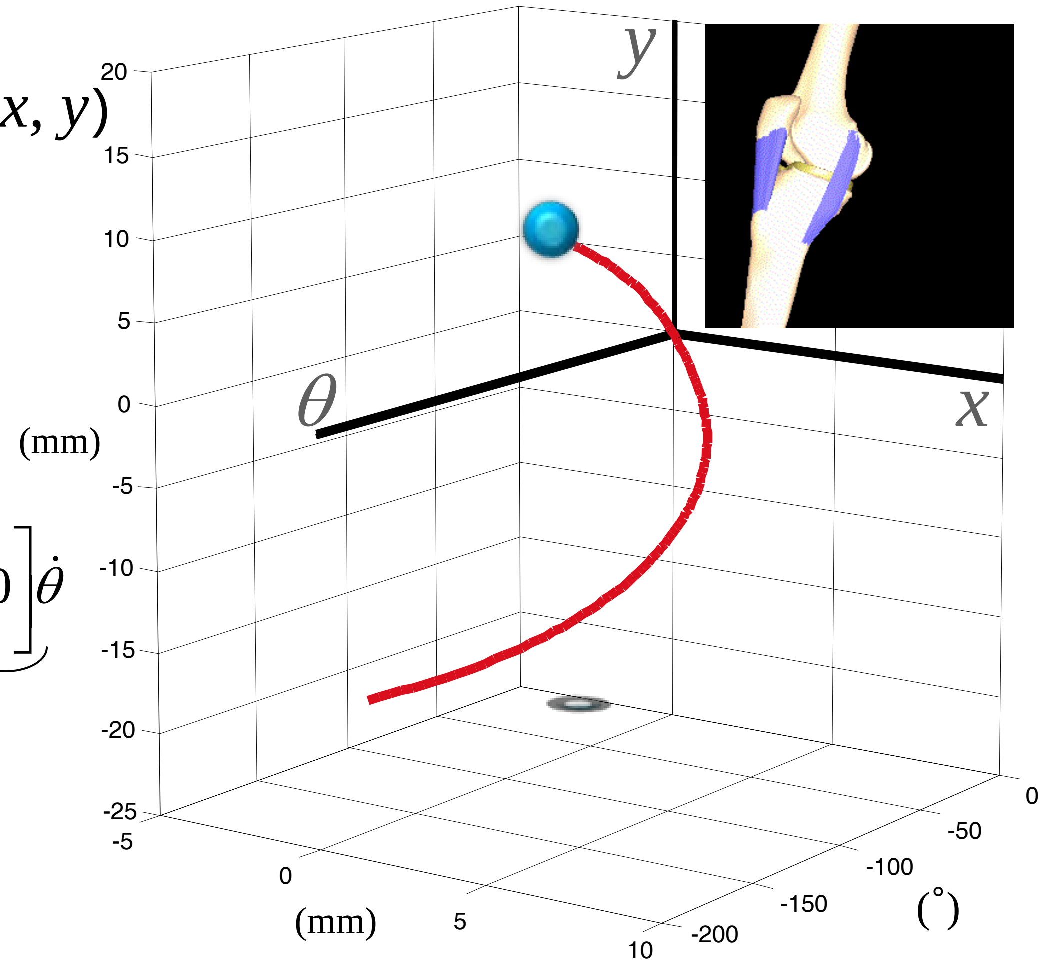
$$^{P}X(\theta)^{B} = \begin{bmatrix} [R(\theta)], & \begin{matrix} x(\theta) \\ y(\theta) \\ 0 \end{matrix} \end{bmatrix}$$

$$q = \theta$$

$$^{P}V^{B} = \underbrace{\begin{bmatrix} 0 & 0 & 1 & \frac{\partial x}{\partial \theta} & \frac{\partial y}{\partial \theta} & 0 \end{bmatrix}}_{^{P}\mathbf{H}^{B}} \dot{\theta}$$

$$u = \dot{\theta}$$

$$^{P}A^{B} = {}^{P}\mathbf{H}^{B}\dot{u} + {}^{P}\dot{\mathbf{H}}^{B}u$$



1 DOF

Function Based Mobilizers

Specify transform between parent and child as a function of ***m*** independent coordinates.

$$^{P}X(\mathbf{x})^{C} = \begin{bmatrix} R(x_1, x_2, x_3) & \begin{matrix} x_4 \\ x_5 \\ x_6 \end{matrix} \end{bmatrix} \qquad \mathbf{x}(q) = \begin{Bmatrix} f_1(q_1, q_2, \ldots, q_m) \\ f_2(q_1, q_2, \ldots, q_m) \\ \vdots \\ f_6(q_1, q_2, \ldots, q_m) \end{Bmatrix}$$

- 6 **`functions`**: describe spatial coordinates, $\mathbf{x}(q)$
  - 1-3 specify angles, 4-6 translations
  - At least twice differentiable
- **`coordIndices`** specify which *q*'s each function uses
- **`axes`** (optional) specify an axis for each x_i
  - 1-3 (body-fixed) and 4-6 (in P) must be linearly independent

Function Based Knee Mobilizer

```
// add shank via right knee joint
MobilizedBody::FunctionBased shank(thigh,
Transform(Vec3(0.0020, 0.1715, 0)), tibia,
Transform(Vec3(0.0, 0.1862, 0.0)),
nm, functions, coordIndices);
```

$\boldsymbol{nm} = 1$, one generalized coordinate, $q[0] = \theta$

functions $= \{0, 0, \theta, f_x(\theta), f_y(\theta), 0\}^T$

coordIndices $= \{\{\},\{\},\{0\},\{0\},\{0\},\{\}\}^T$

Alternative Formulations

```
// add shank via right knee joint
MobilizedBody::FunctionBased shank(thigh,
Transform(Vec3(0.0020, 0.1715, 0)), tibia,
Transform(Vec3(0.0, 0.1862, 0.0)),
nm, functions, coordIndices, axes);
```

$$\mathbf{nm} = 1$$

$$\texttt{functions} = \{\theta, 0, 0, f_x(\theta), f_y(\theta), 0\}^T$$

$$\texttt{coordIndices} = \{\{0\},\{\},\{\},\{0\},\{0\},\{\}\}^T$$

$$\texttt{axes} = \left[\begin{Bmatrix}0\\0\\1\end{Bmatrix},\begin{Bmatrix}1\\0\\0\end{Bmatrix},\begin{Bmatrix}0\\1\\0\end{Bmatrix},\begin{Bmatrix}1\\0\\0\end{Bmatrix},\begin{Bmatrix}0\\1\\0\end{Bmatrix},\begin{Bmatrix}0\\0\\1\end{Bmatrix}\right]$$

Exercise: Create a Knee Mobilizer

1. Compile and run **KneeJointExample.cpp**
2. Convert **shank** type: **Pin** to **FunctionBased**
   - See **MobilizedBody.h**
   - **nm**, **functions** and **coordIndices** are given
   - **fx Spline** is given, **fy** set as **Constant**
3. Scale the **kneeX** translations by 10 to exaggerate the coupled translation.
4. Add a Spline for the Y-direction (**fy**)
   - NOTE: Y translation with respect to thigh origin.

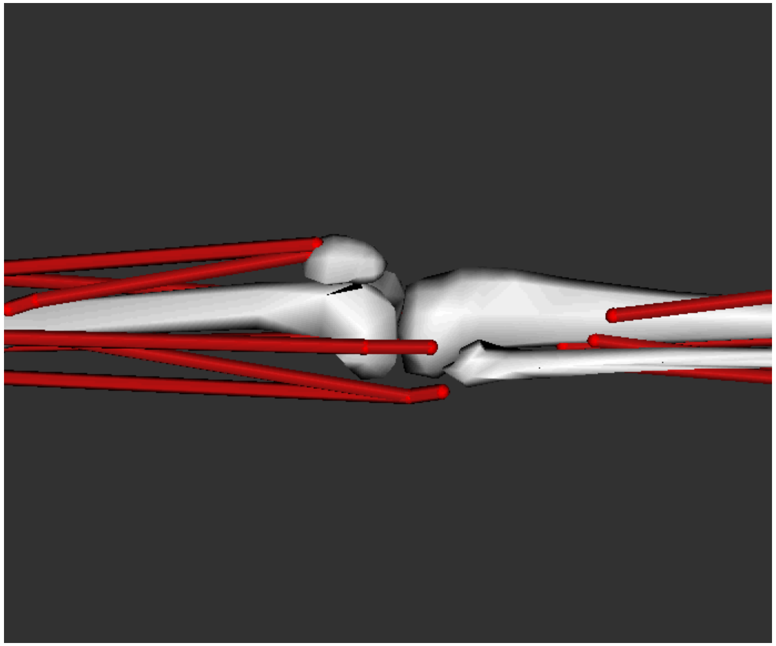
Still just 1 dof!

Mobilized Body

```
// Add a mobilzed body to the system
MobilizedBody::FunctionBased(
      MobilizedBody <parent>,
      Transform <frameOnParent>,
      Body <theBody>,
      Transform <frameOnChild>,
      int <numMobilities>,
      std::vector<const Function<1>*> &functions,
      std::vector<std::vector<int> > &coordIndices
      );
```

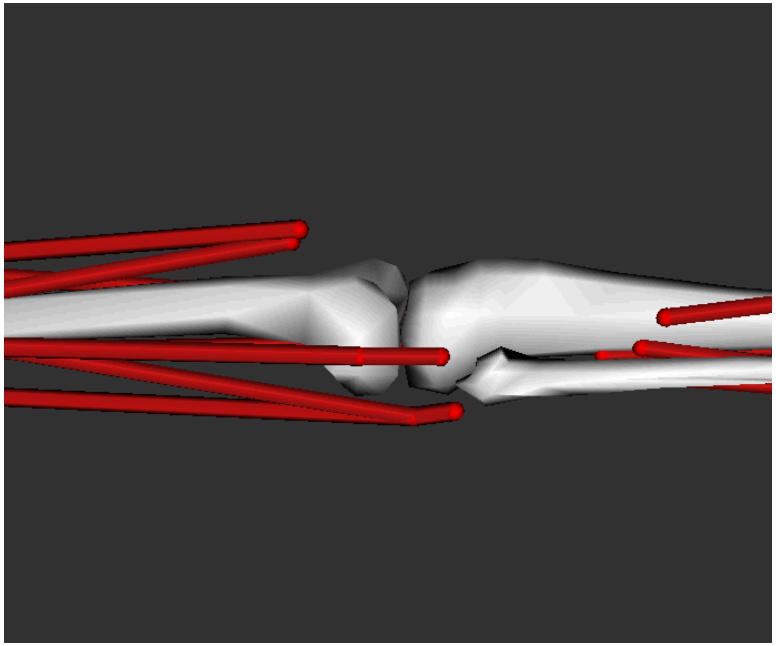
Knee Modeling Comparison

Constraint Enforced Joint



- 3-DOF+2-Constraints = 5 DAEs
- W/ patella: 11 DAEs

Constraint Free Mobilizer



- 1-DOF+0-Constraints = 1 ODE
- W/ moving muscle points = 1 ODE!
- Lose inertial effects of patella

Modeling a Passive Dynamic Walker in Simbody

Eric Lew

What is a Passive Dynamic Walker

- A bipedal machine that naturally walks down a shallow incline.
  - No motors
  - No sensors
- Video from Working Model

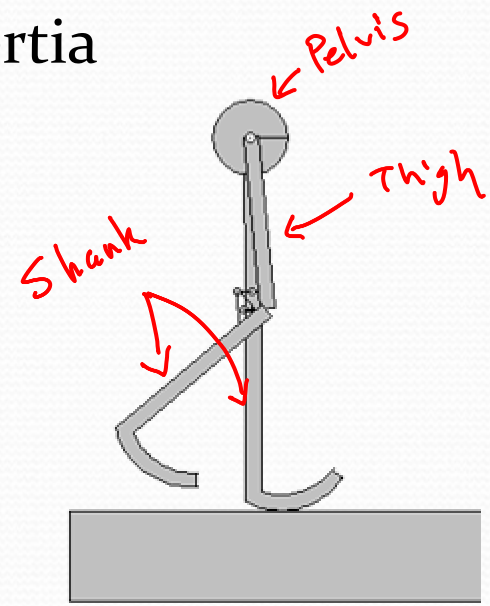
Figure from Collins et al. 2005.

What can we learn from PDWalkers?

- Unlike complex models, they can be easily modified and analyzed to answer specific hypotheses about the role of morphology (vs. neural control) in walking.
  - Kuo 1999 - *Stabilization of Lateral Motion in Passive Dynamic Walking*
    - PDWalkers are inherently unstable in the lateral direction, suggesting that more feedback control is necessary.
    - Follow up study in 2000 showed greater increase in lateral foot placement variability vs. fore-aft variability (53%-21%) when the eyes were closed.

Constructing a Passive Dynamic Model

- Things taken straight from Working Model simulation by Ruina et al.:
  - Geometry
  - Initial Conditions
  - Moments of Inertia



Video

What's missing?

What do we need to implement?

- Knee catch mechanism
- Contact model
    - Friction
    - Normal Force

Knee Catch Mechanism

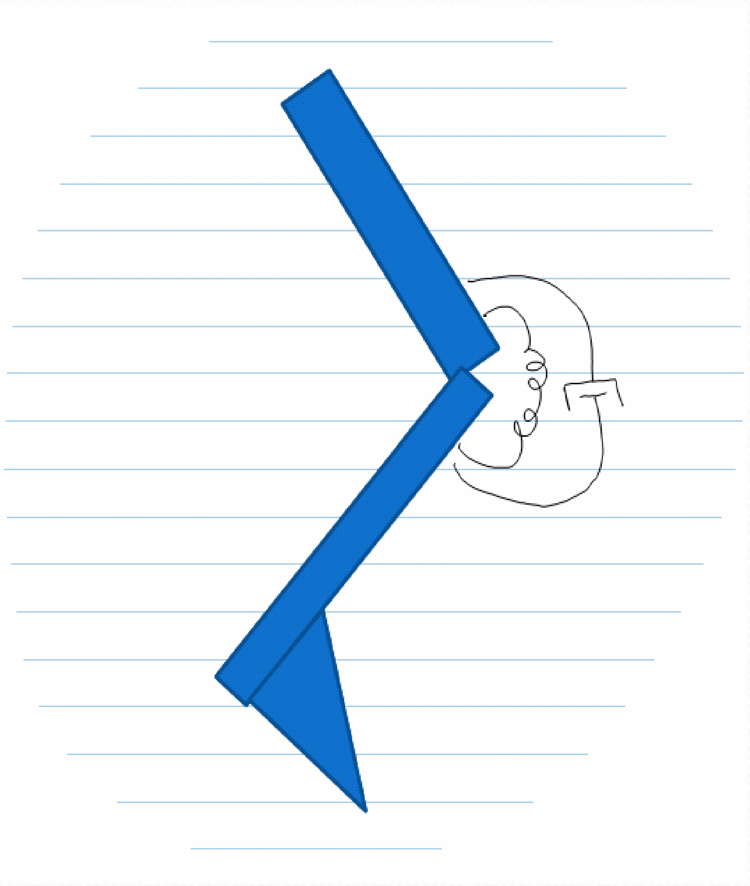
- Behavior of knee catch mechanism
  - Stop tibia when knee reaches 180 degrees.
  - No bounce back (inelastic collision).
  - Release knee when opposite foot makes heel strike.

How can we implement a catch mechanism at the knee within Simbody?

Method #1

Spring Damper System

- Use custom forces
- Problem:
  - Catch mechanism is an inelastic collision
  - Spring alone conserves energy
    - Bounce back
  - Strong damper removes energy
    - Stiff equations of motion.

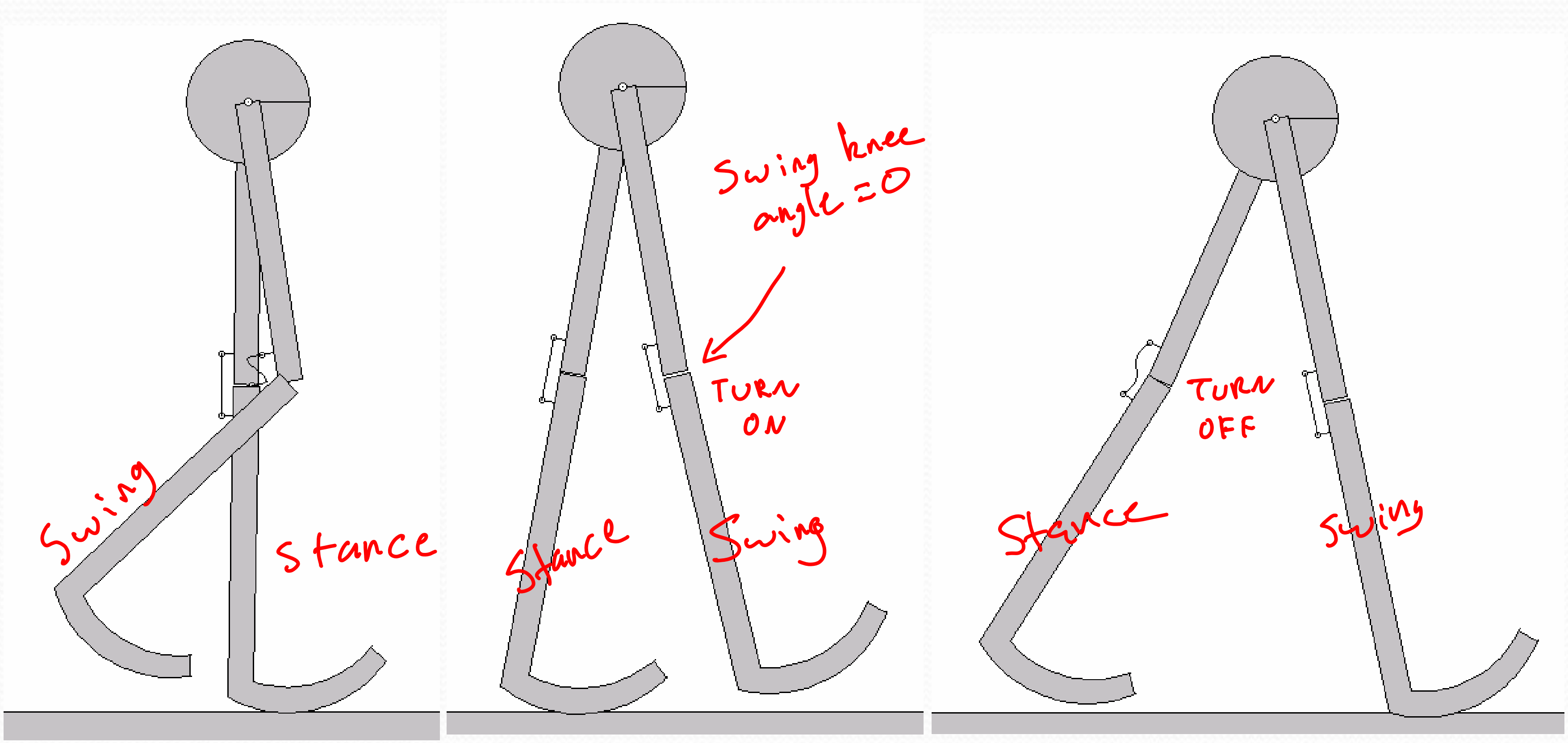


Method #2

Constraints and Event Handlers

- In Simbody, constraints can be turned on and off mid-simulation.
- Use **Event Handlers** to toggle constraints at user-defined times.
- ConstantAngle constraint between shank and thigh locks the knee.

Knee Constraint

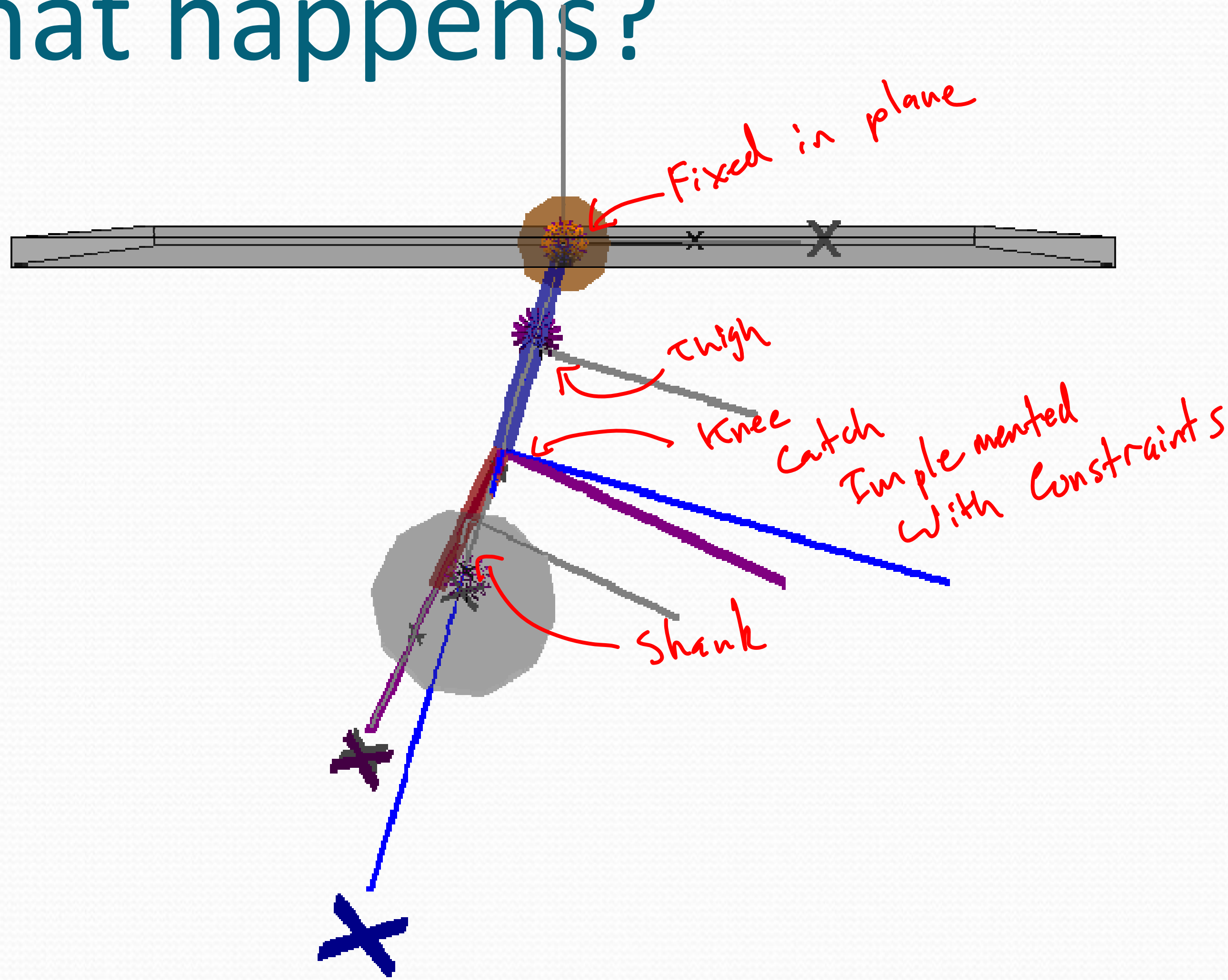


Swing Leg: Constraint OFF
Stance Leg: Constraint ON

Swing Leg: Constraint ON
Stance Leg: Constraint ON

Swing Leg: Constraint ON (heel strike)
Stance Leg: Constraint OFF (toe-off)

So what happens?



$$ {}^{N}H^{S/O} = \sum r^{B_i/O} \times {}^{N}L^{B_i} $$
$$ = \sum \vec{r}^{B_i/O} \times \left(\dot{\vec{r}}^{B_i/O} \cdot m^{B_i} \right) $$

$$ \dot{\vec{r}}^{B_i/O} = {}^{N}\vec{v}^{B_i} = {}^{N}\vec{v}^{\,Bcm} + {}^{N}\vec{\omega}^{B} \times \vec{r}^{\,B_i/Bcm} $$

$$ {}^{N}\vec{H}^{B/O} = \sum m^{B_i} \cdot \vec{r}^{B_i/O} \times \left({}^{N}\vec{v}^{\,Bcm} + {}^{N}\vec{\omega}^{B} \times \vec{r}^{\,B_i/Bcm} \right) $$
$$ = \sum m^{B_i} \cdot \left(\vec{r}^{\,B_i/Bcm} + \vec{r}^{\,Bcm/O} \right) \times {}^{N}\vec{v}^{\,Bcm} + \sum m^{B_i} \cdot \vec{r}^{B_i/O} \times \left({}^{N}\vec{\omega}^{B} \times \vec{r}^{\,B_i/Bcm} \right) $$
$$ + \sum m^{B_i} \cdot \left({}^{N}\vec{\omega}^{B} \cdot \left(\vec{r}^{B_i/O} \cdot \vec{r}^{\,B_i/Bcm} \right) \right) - $$

$$ {}^{G}\vec{H}^{th/O} = \vec{r}^{\,thcm/O} \times m_{th}\, {}^{N}\vec{v}^{\,thcm} + I^{th/thcm} \cdot {}^{N}\vec{\omega}^{th} $$
$$ {}^{N}\vec{v}^{\,thcm} = \underbrace{{}^{N}\vec{v}^{\,pelvis}}_{\text{constant}} + {}^{N}\vec{\omega}^{th} \times \vec{r}^{\,thcm/pelvis} $$

$$ {}^{G}\vec{H}^{th/O} = \vec{r}^{\,thcm/O} \times m_{th} \left({}^{N}\vec{v}^{\,pelvis} + {}^{N}\vec{\omega}^{th} \times \vec{r}^{\,thcm/pelvis} \right) + I^{th/thcm} \cdot {}^{N}\vec{\omega}^{th} $$
$$ = m_{th} \cdot \vec{r}^{\,thcm/O} \times {}^{N}\vec{v}^{\,pelvis} + \vec{r}^{\,thcm/O} \times \left({}^{N}\vec{\omega}^{th} \times \vec{r}^{\,thcm/pelvis} \right) + I\vec{\omega} $$
$$ {}^{N}\vec{\omega}^{th} \left(\vec{r}^{\,thcm/O} \cdot \vec{r}^{\,thcm/pelvis} \right) - \vec{r}^{\,thcm/pelvis} \left(\vec{r}^{\,thcm/O} \cdot {}^{N}\vec{\omega}^{th} \right) + I\vec{\omega} $$

$$ {}^{N}\vec{H}^{th/O} = m_{th}\, \vec{r}^{\,thcm/O} \times {}^{N}\vec{v}^{\,thcm} + I^{th/thcm} \cdot {}^{N}\vec{\omega}^{th} $$
$$ = m^{th}\, \vec{r}^{\,thcm/O} \times \left({}^{N}\vec{v}^{\,pelvis} + {}^{N}\vec{\omega}^{th} \times \vec{r}^{\,thcm/pelvis} \right) + I^{th/thcm} \cdot {}^{N}\vec{\omega}^{th} $$
$$ = m^{th}\, \vec{r}^{\,thcm/O} \times {}^{N}\vec{v}^{\,pelvis} + \vec{r}^{\,thcm/O} \times \left({}^{N}\vec{\omega}^{th} \times \vec{r}^{\,thcm/pelvis} \right) m^{th} + I^{th/thcm} \cdot {}^{N}\vec{\omega}^{th} \rightarrow 0 $$
$$ = m^{th}\, \vec{r}^{\,thcm/O} \times {}^{N}\vec{v}^{\,pelvis} + {}^{N}\vec{\omega}^{th} \left(\vec{r}^{\,thcm/O} \cdot \vec{r}^{\,thcm/pelvis} \right) - \vec{r}^{\,thcm/pelvis} \left(\vec{r}^{\,thcm/O} \cdot {}^{N}\vec{\omega}^{th} \right) + I^{th/thcm} \cdot {}^{N}\vec{\omega}^{th} $$
$$ {}^{N}\vec{H}^{th/O} = m^{th}\, \vec{r}^{\,thcm/O} \times {}^{N}\vec{v}^{\,pelvis} + {}^{N}\vec{\omega}^{th} \left(\vec{r}^{\,thcm/O} \cdot \vec{r}^{\,thcm/pelvis} \right) m^{th} + I^{th/thcm} \cdot {}^{N}\vec{\omega}^{th} $$
$$ {}^{N}\vec{H}^{th/O -} = {}^{N}\vec{H}^{th/O +} $$

$$ {}^{N}\vec{v}^{\,pelvis -} = {}^{N}\vec{v}^{\,pelvis +} $$

all pelvis center constant

$$ {}^{N}\vec{\omega}^{th +} = {}^{N}\vec{\omega}^{th -} + \Delta\vec{\omega} $$

$$ m^{th}\, \vec{r}^{\,thcm/O} \times {}^{N}\vec{v}^{\,pelvis} + {}^{N}\omega^{th} \left[\vec{r}^{\,thcm/O} \cdot \vec{r}^{\,thcm/pelvis} \cdot \underset{\text{identity matrix}}{\mathbb{1}} + I^{th/thcm} \right] = \left({}^{N}\vec{\omega}^{th} + \Delta\vec{\omega} \right) \left[\vec{r}^{\,thcm/O} \cdot \vec{r}^{\,thcm/pelvis} \, \mathbb{1} + I^{thcm/thcm} \right] + m^{th}\, \vec{r}^{\,thcm/O} \times {}^{N}\vec{v}^{\,pelvis} + $$

$$ {}^{N}\vec{H}^{T/O} = \vec{r}^{\,Tcm/O} \times m_T\, {}^{N}\vec{v}^{\,Tcm} + I_N^{T/Tcm} \cdot {}^{N}\vec{\omega}^{T} $$
$$ = m_T\, \vec{r}^{\,Tcm/O} \times \left({}^{N}\vec{v}^{\,pelvis} + {}^{N}\vec{\omega}^{T} \times \vec{r}^{\,Tcm/pelvis} \right) + I_N^{T/Tcm} \cdot {}^{N}\vec{\omega}^{T} $$
$$ = m_T\, \vec{r}^{\,Tcm/O} \times {}^{N}\vec{v}^{\,pelvis} + m_T\, \vec{r}^{\,Tcm/O} \times \left({}^{N}\vec{\omega}^{T} \times \vec{r}^{\,Tcm/pelvis} \right) + I^{T/Tcm} \cdot {}^{N}\vec{\omega}^{T} $$
$$ {}^{N}_{N}\vec{H}^{T/O} = m_T\, \vec{r}^{\,Tcm/O} \times {}^{N}\vec{v}^{\,pelvis} + I^{T/Tcm} \cdot {}^{N}\vec{\omega}^{T} + m_T\, {}^{N}\vec{\omega}^{T} \left(\vec{r}^{\,Tcm/O} \cdot \vec{r}^{\,Tcm/pelvis} \right) - m_T\, \vec{r}^{\,Tcm/pelvis} \left(\vec{r}^{\,Tcm/O} \cdot {}^{N}\vec{\omega}^{T} \right) \rightarrow 0 $$

$$ {}^{N}\vec{H}^{T/O -} = m_T\, \vec{r}^{\,Tcm/O} \times {}^{N}\vec{v}^{\,pelvis} + \left(I_N^{T/Tcm} + m_T\, \vec{r}^{\,Tcm/O} \cdot \vec{r}^{\,Tcm/pelvis} \cdot \mathbb{1} \right) \cdot {}^{N}\vec{\omega}^{T} $$

$$ \underbrace{{}^{N}\vec{H}^{T/O -} + {}^{N}\vec{H}^{S/O -}}_{\text{From SimBody}} = {}^{N}\vec{H}^{TS/O +} = m_{TS}\, \vec{r}^{\,TScm/O} \times {}^{N}\vec{v}^{\,pelvis} + \left(I^{TS/TScm} + m_{TS}\, \vec{r}^{\,TScm/O} \cdot \vec{r}^{\,TScm/pelvis} \cdot \mathbb{1} \right) \cdot {}^{N}\vec{\omega}^{TS} $$

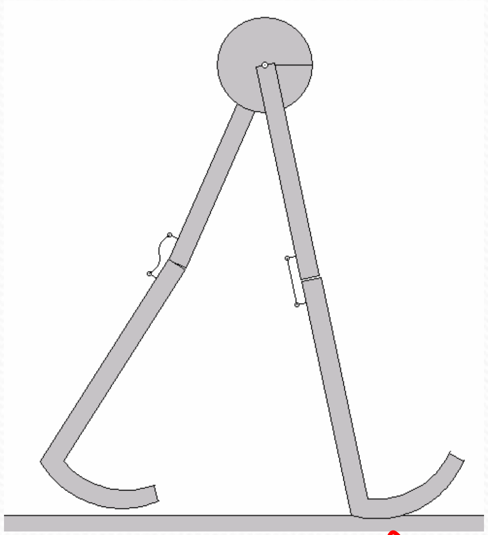
$$ {}^{N}\vec{\omega}^{TS} = \left(I^{TS/TScm} + m_{TS}\, \vec{r}^{\,TScm/O} \cdot \vec{r}^{\,TScm/pelvis} \cdot \mathbb{1} \right)^{-1} \cdot \left({}^{N}H^{T/O -} + {}^{N}\vec{H}^{S/O -} - m_{TS}\, \vec{r}^{\,TScm/O} \times {}^{N}\vec{v}^{\,pelvis} \right) $$

Contact Model

- Requirements for Contact Model
  - Provide appropriate normal force
  - Provide friction force so foot will roll without slipping

Method #1

- Use a constraint to implement normal force

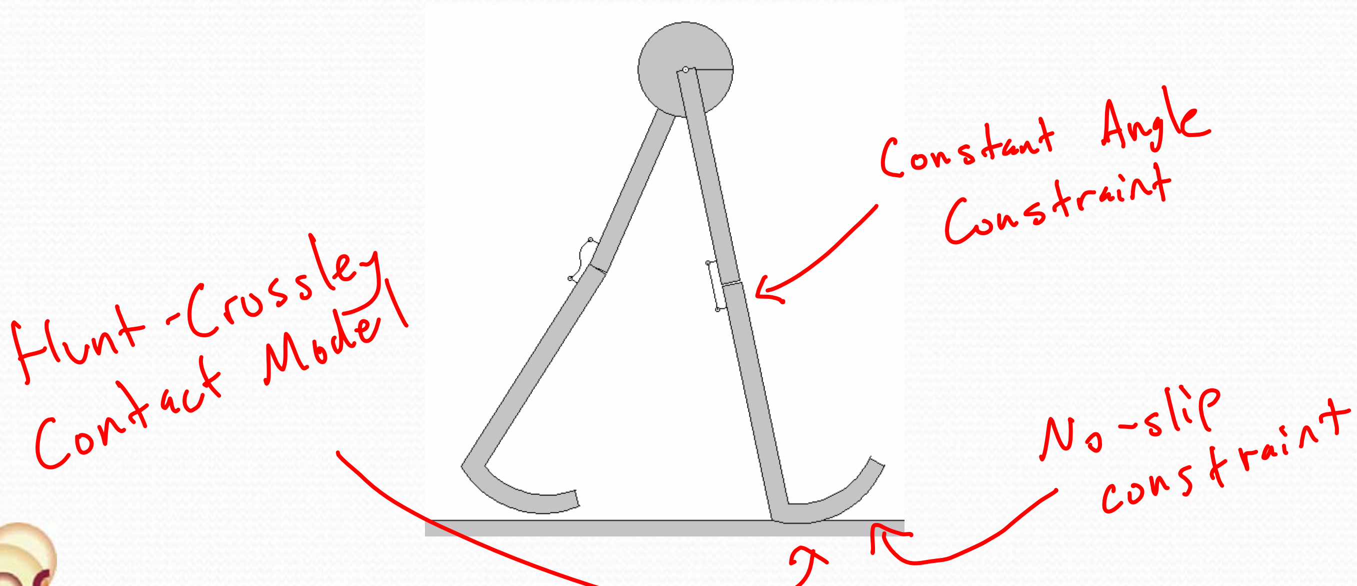


Roll w/o slip constraint
Point-in-plane constraint

- Angular momentum problem again

Method #2

- Use Hunt-Crossley Contact Model to implement normal force.



Future Work

- Optimize code to run in real time
- Define specific, testable hypothesis
- Be able to generate new limit cycles for different geometries

Exercises

- Compile code and run
- Try two different materials for Hunt Crossley model and run

Acknowledgements

- Scott Delp
- Peter Eastman
- Samuel Hamner
- Jeff Reinboldt
- Ajay Seth

References

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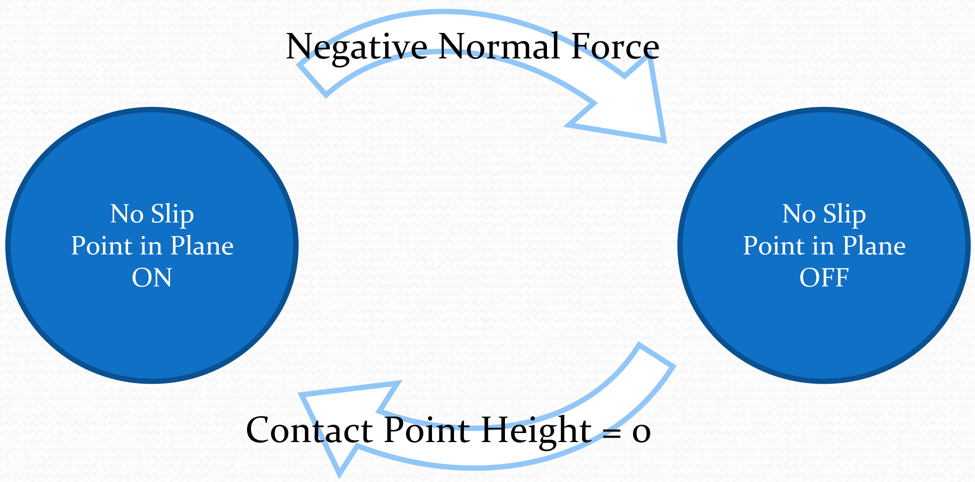
Kurz, M. J., Judkins, T. N., Arellano C., Scott-Pandorf M., (2008). A passive dynamic walking robot that has a deterministic nonlinear gait, 41, 1310-1316.

Questions?

Approach #1

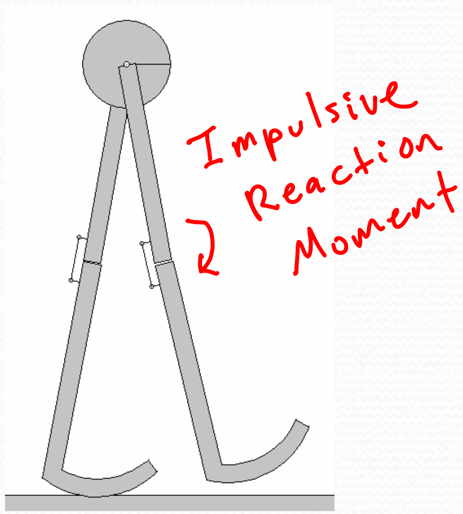
- Use a point-in-plane constraint to keep the foot contact point on the ground
- Use a no-slip constraint to provide friction force



Angular Momentum Again

Knee Constraint

- Impulsive moment does **not** get transmitted to other segments.
  - Pin joints transmit no moment
- No External Force



Contact Constraint

- Impulsive force **does** get transmitted to other segments.
- External Force